Work and Energy

The frustration you sometimes feel when doing homework is a good thing. It lets you know where your understanding is incomplete.

Suppose you drop a 1-kg rock from a height of 5 m above the ground.When it hits, how much force does the rock exert on the ground?

1. 0.2 N
2. 5 N
3. 10 N
4. 50 N
5. 100 N
6. 500 N
7. impossible to determine

A 1 kg ball is dropped and falls 1 meter. The work done by the Earth's gravitational force on the ball is approximately:

1. 0 J
2. $+10$ J
3. -10 J
4. none of the above

ANS: **2**—Gravity does $+10\,\text{J}$ of work on the ball.

The gravitational force (weight) on the ball is approximately $10\,\text{N}$ downward. The displacement of the ball is 1m downward. Because the force and displacement are parallel, gravity does positive work: $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = F\,s\,\cos 0^\circ \;= +10\text{J}$.

A $1\,\text{kg}$ ball is thrown upward and rises a height of $1\,\text{meter}$. The work done by the Earth's gravitational force on the ball is approximately:

1. $0\,\text{J}$
2. $+10\,\text{J}$
3. $-10\,\text{J}$
4. none of the above

ANS: **3**—Gravity does $-10\,\mathrm{J}$ of work on the ball.

The gravitational force (weight) on the ball is approximately $10\,\mathrm{N}$ downward. The displacement of the ball is $1\,\mathrm{m}$ *upward*. Because the force and displacement are anti-parallel, gravity does *negative* work: $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = F\,s\cos 180^\circ = -10\,\mathrm{J}$.

A $1\,\text{kg}$ ball is thrown upward and rises a height of $1\,\text{meter}$. The change in the ball's kinetic energy during this rise is approximately:

1. $0\,\text{J}$
2. $+10\,\text{J}$
3. $-10\,\text{J}$
4. none of the above

ANS: **3**—The change in the ball's KE is $-10\,\mathrm{J}$.

The change in the ball's kinetic energy is equal to the net work done on it. In this case, the only force on it is the (downward) gravitational force, which does $-10\,\mathrm{J}$ of work. Therefore, $\Delta K = -10\,\mathrm{J}$.

A 1 kg ball is initially on the ground. Your professor lifts the ball and places it at rest on a table 1 meter high. The work done by the Earth's gravitational force on the ball is:

1. zero
2. + 10 Joules
3. - 10 Joules
4. none of the above

A 1 kg ball is initially on the ground. Your professor lifts the ball and places it at rest on a table 1 meter high. The work done by the professor on the ball is:

1. zero
2. + 10 Joules
3. - 10 Joules
4. none of the above

Your professor lifts a 1 kg ball from the ground and places it at rest on a table $\Delta h = 1$ m high. The work done by the professor on the ball is 10 Joules. Which of the following are valid explanations of that result?

1. $W_{prof} = F_{prof} \Delta h$
2. $\Delta K_t = 0$, so $W_{total} = 0$
3. Both of the above
4. None of the above

Your professor now moves the ball 1 meter farther away on the same table and sets it there. The work done by the professor on the ball is:

1. zero
2. + 10 Joules
3. - 10 Joules
4. none of the above

A 1 kg ball is released from rest and allowed to roll down a 2 m long ramp that makes an angle of 30° above the horizontal. The work done by the Earth's gravitational force on the ball is approximately:

1. $+10$ J
2. $+20$ J
3. -10 J
4. -20 J
5. none of the above

ANS: **1**—Gravity does $+10\,\mathrm{J}$ of work on the ball.

The gravitational force (weight) on the ball is approximately $10\,\mathrm{N}$ downward. The displacement of the ball is $2\,\mathrm{m}$ down and to the left. In fact, the force and displacement vectors make a $60°$ angle with respect to each other. The work done by gravity, therefore, is $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = (10\,\mathrm{J})(2\,\mathrm{m})\cos 60° = +10\mathrm{J}$.

We can also look at these vectors in component form. First, let's break the force vector up into components parallel and perpendicular to the surface. (This is our old friend, the "inclined plane" problem.) Define the *x* direction to be down slope, and the *y* direction to be down, perpendicular to the surface. Then the gravitational force has components $F_x = F\sin 30°$, and $F_y = F\cos 30°$. Therefore, the gravitational force, expressed in component form, is $\vec{\boldsymbol{F}} = (F\sin 30°)\,\hat{\boldsymbol{\imath}} + (F\cos 30°)\,\hat{\boldsymbol{\jmath}}$. The displacement vector has magnitude $s = 2\,\mathrm{m}$, and is directed down-slope, so $\vec{\boldsymbol{s}} = s\,\hat{\boldsymbol{\imath}}$. Therefore, the work done is

$$W = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = (F\sin 30°)(s) + (F\cos 30°)(0) = Fs\sin 30°$$

so

$$W = (10\,\mathrm{N})(2\,\mathrm{m})(1/2) = +10\,\mathrm{J}\,.$$

Try This: You can also resolve the two vectors into horizontal and vertical components. This is no work for the gravitational force, but is work for the displacement vector. Try it and prove to yourself that you get the same result.

A 1kg ball is released from rest and allowed to roll down a 2m long ramp that makes an angle of 30° above the horizontal. The work done by the surface of the ramp on the ball is approximately:

1. +10J
2. +20J
3. -10J
4. -20J
5. none of the above

Warmup Question

Discuss the work a pitcher exerts on a baseball as he throws a fast ball. Over approximately what distance is the force exerted? Please explain your reasoning thoroughly.

ANS: We could estimate the work done if we knew the force the pitcher applies to the ball and the distance through which the force is applied. We could try to figure that out, but it's actually easier to approach it from a work-energy standpoint.

If the pitching force is the only force that does work (I will ignore any work done by gravity here), then the work done in pitching the ball will equal the change in the ball's kinetic energy. The mass of a baseball is around 0.150 kg, while the speed of a fast ball is around 100 miles/hour, or 45 m/s. This gives a kinetic energy of $\frac{1}{2}mv^2 \approx 150\,\mathrm{J}$. This will be equal to the work done by the throwing force. The force is applied through approximately 1 m of displacement, so the average throwing force would be around 150 N.

Warmup Question

Estimate how much work (in joules) is done by gravity when you roll out of bed and fall to the floor.

ANS: My mass is 100 kg, so my weight is $mg \approx (100\,\mathrm{kg})(10\,\mathrm{N/kg}) = 1000\,\mathrm{N}$. My bed is roughly 1 m off the floor. Therefore, gravity does $(1000\,\mathrm{N})(1\,\mathrm{m}) = 1000\,\mathrm{J}$ of work. This work is positive because the force and the displacement are parallel.

Warmup Question

While a skateboarder coasts down the curved side of a half pipe (see below), does the normal force exerted by the surface on the skateboard do any work?

1. Yes, a positive amount
2. Yes, a negative amount
3. No, the work done is zero

ANS: **3**—The normal force does no work on the skateboard.

The “normal” force is perpendicular to the surface, while the displacement of the skateboard is along the surface. These two vectors are perpendicular. Mathematically the work is the scalar (“dot”) product between the force and the displacement. This product is zero because those two vectors are perpendicular.

We can also relate this to physical concepts. The normal force exerted by the surface on the skateboard is perpendicular to the motion. Therefore, it can contribute to the radial (direction changing) force on the skateboard, but not the tangential (speed changing) force. Because the normal force will not cause the skateboard’s speed to change, it cannot contribute to any change in kinetic energy. Therefore, the force does no work.