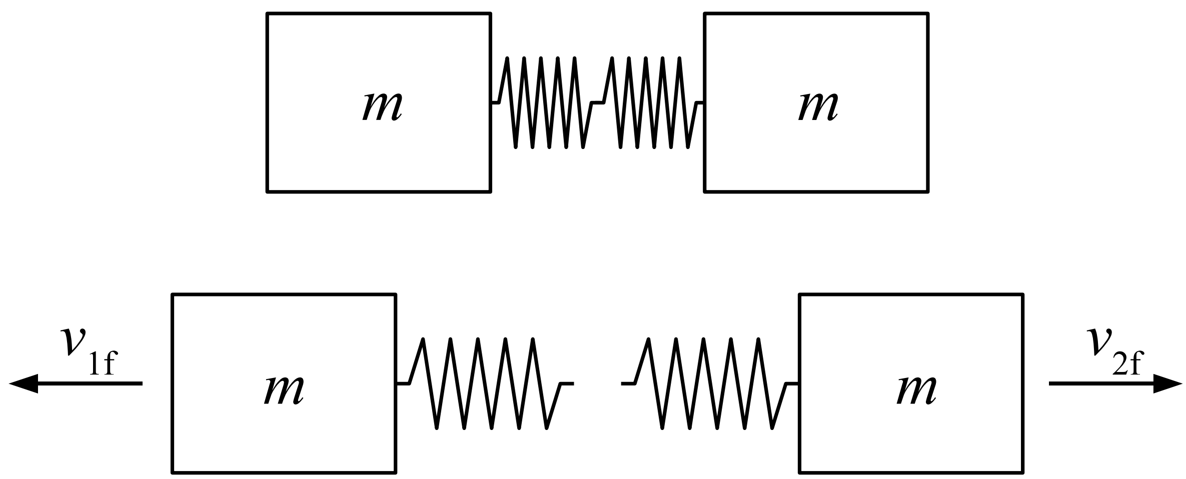
Potential Energy and Conservation

The average college student in 1960 devoted 40 hours a week to academic matters. Today, it’s down to 27 hours. That gives you a tremendous advantage relative to your peers if you make the most of the time available to you.

Two blocks of equal mass are attached to springs and held together until some instant when they are abruptly released. Which of the following quantities are of equal magnitude?



1. The force each block exerts on the other at each instant
2. The amount of work each block performs on the other
3. The kinetic energies of the blocks after they have separated
4. All of the above
5. 1 & 3, but not 2

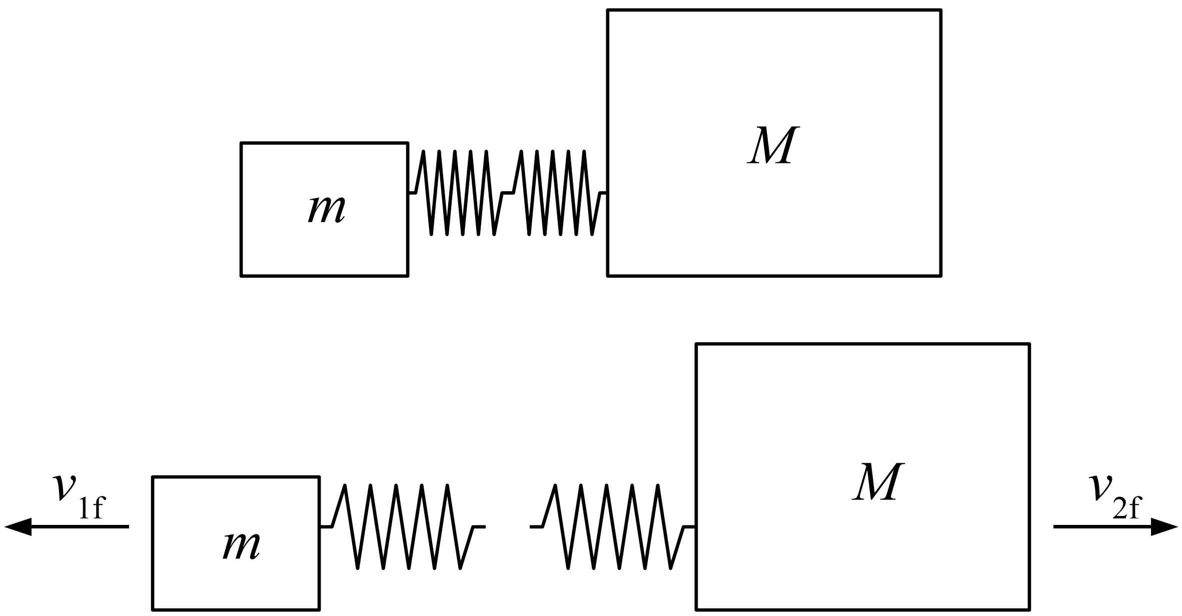
ANS: **4**—Choices 1–3 are all correct.

We know that the blocks exert equal forces on each other from Newton's third law. This is always true for forces in an interaction.

To see that the other answers are also correct, note that the diagram is completely symmetric. The masses identical, which means (because of the forces are equal) that the accelerations are equal. These masses are in contact for the same total time, meaning that they have the same final speed and therefore the same kinetic energy. The kinetic energy gained by each mass is equal to the work done on it by the other mass, so the work done by each mass on the other is also the same. (Note that there was no reference to the springs being identical. It does not matter, so long as masses of the blocks (including springs) are equal.)

This question is even easier to answer by considering conservation of momentum. The net initial momentum of the system is zero, so the net final momentum of the system will also be zero. Therefore, the blocks will have equal momenta in opposite directions. Because they have equal masses, their velocities will also be equal. From this result we can see that they will have equal kinetic energies ($K = \frac{1}{2}mv^2 = p^2/2m$). From the work-kinetic energy theorem, equal kinetic energies implies equal work done on each block.

Two blocks of very unequal mass are attached to springs and held together until some instant when they are abruptly released. Which of the following quantities are of equal magnitude?



1. The force each block exerts on the other at each instant
2. The amount of work each block performs on the other
3. The kinetic energies of the blocks after they have separated
4. All of the above
5. 1 & 3, but not 2

ANS: **1**—The blocks exert equal forces on each other, according to Newton's third law.

However, they will not have the same kinetic energy and work done. The acceleration of the smaller block will be greater than the acceleration of the larger block, so the smaller block will have a greater final speed and a greater displacement over the time period during which the forces are applied. This means that there will be more work done on the smaller block (same force, greater displacement) than on the larger block. Since they both start from rest, this also means that the smaller block will have a greater final kinetic energy than will the larger block.

Again, this is even easier to answer using conservation of momentum. Because the total initial momentum of the system is zero, the total final momentum of the system will be zero, meaning that the two blocks will have equal and opposite momenta. The larger-mass object will have a smaller final kinetic energy ($K = p^2/2m$), and therefore a smaller work done on it, than the smaller mass.

Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

1. as much kinetic energy as the lighter one.
2. twice as much kinetic energy as the lighter one.
3. half as much kinetic energy as the lighter one.
4. four times as much kinetic energy as the lighter one.
5. impossible to determine

ANS: **2**—The heavier ball has twice as much kinetic energy just before hitting the ground.

We can understand this in terms of the work-kinetic energy theorem. The work done by gravity, $-mg\Delta y$, is twice as large for the heavier ball. The gravitational force is the only force that does work on the balls, so the heavier ball will have twice the change in kinetic energy.

We can also see this by considering conservation of mechanical energy. Gravity is a conservative force, so the total mechanical energy, $E_{\text{tot}} = K + U$, is constant in the problem. The heavier ball has twice the change in potential energy, $mg\Delta y$, so it will have twice the change in kinetic energy.

A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v_1 at the bottom. To achieve speed $v_2 = 2v_1$ at the bottom, how many times as high must a new ramp be?

1. 1
2. $2^{1/2} = 1.414$
3. $3/2 = 1.5$
4. 2
5. 3
6. 4
7. 5

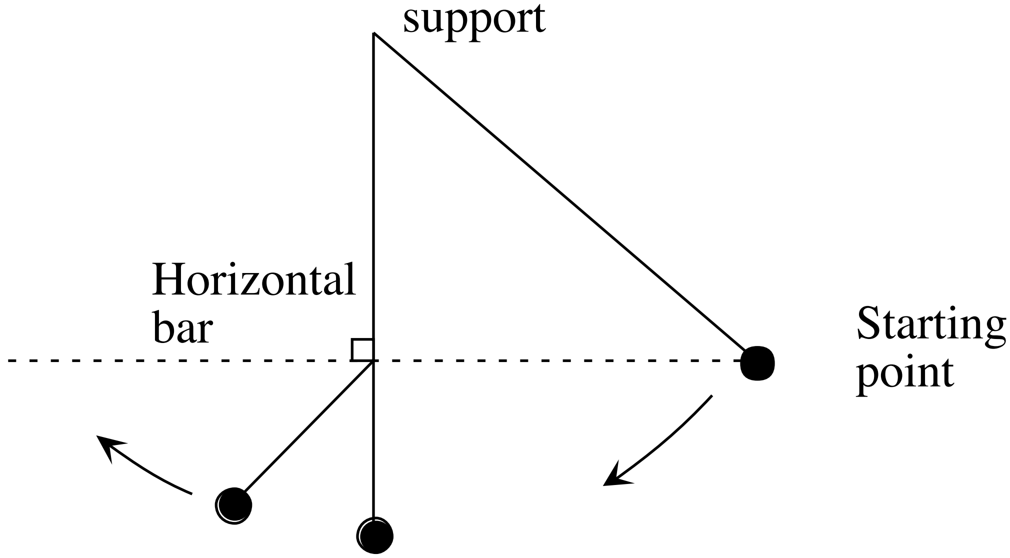
ANS: **4**—The new ramp must be four times as high.

This is answered most easily using conservation of mechanical energy because there are no dissipative forces in the problem. The second block is to achieve twice the final velocity, and hence four times the final kinetic energy ($K = \frac{1}{2}mv^2$), of the first block. Therefore, the second block must have four times the change in potential energy. This will happen if we allow the second block to fall four times the distance of the first block.

A rubber bottle stopper swings around in a vertical plane at the end of a cord attached to a fixed pivot point. From the top of the circle to the bottom, the work done by the cord on the stopper is

1. positive
2. negative
3. zero
4. more information is needed

A heavy knob swings from a string attached to a support, as shown below. When the string reaches the vertical position, it is stopped by a horizontal bar level with the knob's starting point. The part below continues swinging. How high does the knob swing before stopping?



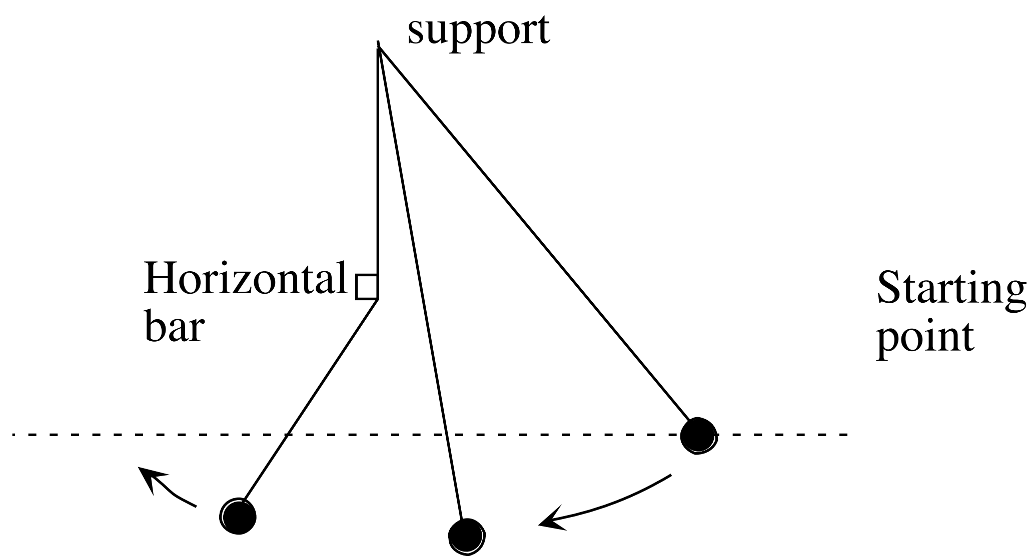
1. Higher than the starting level
2. At the starting level
3. Below the starting level
4. Need more information

ANS: **2**—The knob swings up to to the bar's level.

This is most easy to see in terms of mechanical energy. There are only two forces on the knob: gravity, and the tension force due to the string. Gravity is a conservative force, so it will not cause the mechanical energy to change. The tension force does no work on the knob at all, so it will not affect the mechanical energy. Therefore, the mechanical energy is the sum of the kinetic and potential energies and is conserved. The starting point has $v = 0$ and therefore $K = 0$. At this point, the potential energy is maximum, equal to the total mechanical energy.

The question asks for the highest point the knob will reach. This will again correspond to a point where $K = 0$ and potential energy is equal to the total (conserved) mechanical energy. This will be the original starting height of the knob, which is the level of the bar.

A heavy knob swings from a string attached to a support, as shown below. When the string reaches the vertical position, it is stopped by a horizontal bar, this time **above** the knob's starting point. The part below the bar continues swinging. How high does the knob swing before stopping?

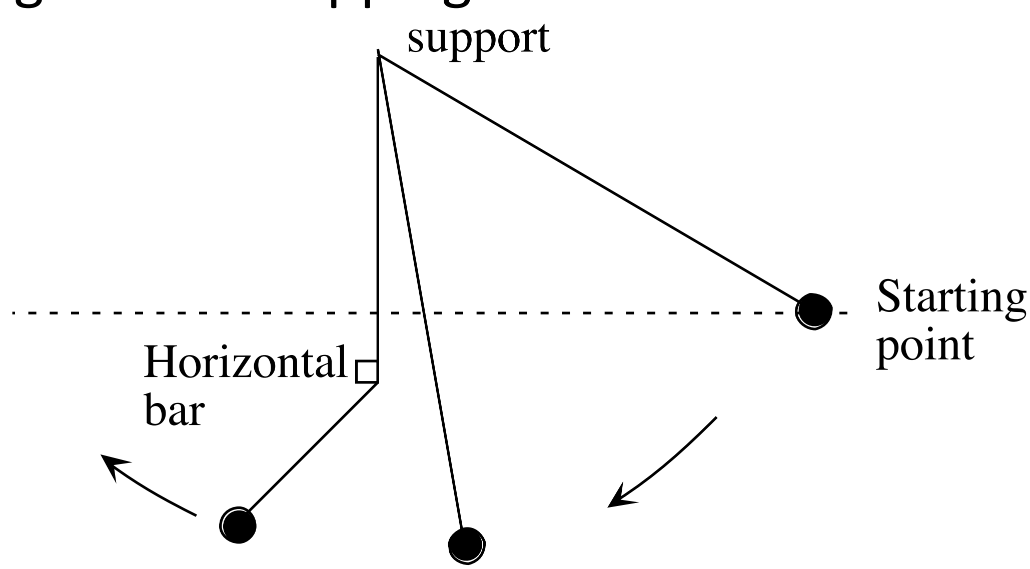


1. Higher than the starting level
2. At the starting level
3. Below the starting level
4. Need more information

ANS: **3**—The knob will swing up to a height below the bar’s level.

Because of conservation of mechanical energy, the knob will only rise as high as its original height, which is below the bar’s level.

A heavy knob swings from a string attached to a support, as shown below. When the string reaches the vertical position, it is stopped by a horizontal bar, this time **below** the knob's starting point. The part below the bar continues swinging. How high does the knob swing before stopping?



1. Higher than the starting level
2. At the starting level
3. Below the starting level
4. Need more information

Warmup Question

As a skydiver falls at constant terminal velocity, does her total mechanical energy (potential energy plus kinetic energy) remain constant? Why or why not? Is total energy conserved in this situation? Explain.

ANS: Mechanical energy does not remain constant because there is a non-conservative (dissipative) force of air resistance on her. This force does negative work on her, removing mechanical energy from the system that consists of her and Earth.

At terminal speed, kinetic energy remains constant. However, she continues to fall, so her gravitational potential energy will decrease. The mechanical energy, the sum of kinetic and gravitational potential energies, therefore decreases.

Total energy is, of course, conserved. That means that the gravitational potential energy lost as she continues to fall is converted into some other forms, including mainly thermal energy.

Warmup Question

Reminder: $1\,\text{watt} = 1\,\text{joule/s}$. First, estimate the amount of kinetic energy in your car when it is moving at highway speed. Then figure out how long it would take a $100\,\text{W}$ light bulb to supply that energy. (Note that cars do not typically operate this way.)

ANS: Kinetic energy is expressed as $\frac{1}{2}mv^2$, where m is the mass of the car and v is its speed. My car weighs around $1000\,\text{kg}$. Highway speed is approximately $30\,\text{m/s}$. Therefore, my car's kinetic energy would be $\frac{1}{2}(1000\,\text{kg})(30\,\text{m/s})^2 = 450000\,\text{J}$. A light bulb uses energy at a rate of $100\,\text{J/s}$, so it would take $(450000\,\text{J})/(100\,\text{J/s}) = 4500\,\text{s} = 1.25$ hours to use that much energy.

Warmup Question

Which of the following doesn't involve a conservative force?

1. Friction
2. Gravity
3. Springs
4. Republicans
5. Tories

ANS: **1**—Friction is not a conservative force.

Gravitational and spring (elastic) forces are both conservative. Republicans and Tories are two groups who consider themselves conservatives, so insofar as members of these groups can exert forces, one might consider them “conservative” (but not, of course, in the physical sense of the word.)