

Old fashion bicycles have fixed-gear drives similar to what is shown below. Rather than turning the wheels themselves, riders turn pedals attached to a toothed disk upon which a chain rides without slipping. The same chain rides on a second, smaller toothed disk on the back tire. Discuss how the chain relates the rotational motion of the two disks to each other. Explain your logic.

**An okay description, but doesn't specify what kind of speed**

The chain connects the two disks together. The middle of each disk is its fixed center or axis of rotation. Each disk rotates around its specific axis of rotation. The chain makes it so that both disks rotate at the same time in the same direction and at the same speed.

**A better description where the kind of speed is specified**

Since we know that when a person peddles a bike it makes the chain pull the back wheel and cause it to rotate in turn giving the bike motion. Therefore, we know that all the disks must have the same angular velocity because they are rotating the same amount over the same period of time.

$$w = d(\text{theta})/dt$$

However, the tangential velocity of the disks would differ, because their radii would be different.

$$v_t = rw$$

Since  $w$  for both disks would be the same the difference in tangential velocity would all depend on the radii. We will call the radius of the larger disk  $R_1$  and the radius of the smaller disk  $R_2$ , same for velocity larger disk is  $V_1$  and smaller disk is  $V_2$ . Because the equation for tangential velocity only uses  $r$  and  $w$ , and  $R_1 > R_2$  we would expect  $V_1$  to be greater than  $V_2$ .

**The actual velocity they have in common is tangential**

Tangential velocity is given by the equation  $v_t = r \cdot \text{angular velocity}$ . The angular velocity is the same for any point on the first disc, however as  $r$  increases so does the tangential velocity of a given point. The chain is attached to the outside of the first disk allowing it to have the greatest possible tangential velocity. The chain then relates this same tangential velocity to the outside of the second disk. However, disk two has a smaller radius. The same tangential velocity with a smaller radius implies a greater angular velocity. In other words disc 2 is spinning faster than disc 1 allowing the bike to go faster.

Estimate the magnitude of the tangential velocity of an object in Birmingham due to the rotation of the Earth. Describe your logic. Your answer should be in m/s. (If you promise not to get too hung up on being precise, the distance from the Earth's pole to the equator is 10,000 km, which originally defined the kilometer.)

**Correct logic, answer kind of awkward**

$$V_t = rw$$

the radius of the earth is 10,000km

$w = \text{change in } \theta / \text{change in time}$

$$w = 2\pi / 1 \text{ day}$$

$$r = 10,000 \text{ km}$$

$$2\pi/\text{day}(10,000 \text{ km}) = 20,000\pi \text{ km/day}$$

**A very good response where the lone error is immediately obvious (check units!)**

The radius of the Earth is 10,000km.

10,000km needs to be in meters, so  $(10,000\text{km})(1,000\text{m}/1\text{km}) = 1 \times 10^7 \text{m}$

It takes 1 day for the Earth to make a full rotation:

$$(1\text{day})(12 \text{ hours}/1\text{day})(3600\text{seconds}/1\text{hour}) = 40,000\text{s}$$

$$v_t = 2(\pi)rt = 2(\pi)(1 \times 10^7 \text{m})(40,000\text{s}) = 2 \times 10^7 \text{m}(40,000\text{s}) \\ = 80,000 \times 10^7 \text{ m/s} = 8 \times 10^{11} \text{m/s}$$

**Another illustration of the importance of units**

Given our formula for tangential velocity:  $v=rw$ . Taking a guess that earth's radius is 6000 km and the tangential velocity of earth's rotation is 8 m/s. we would plug in:  $v=6000 \times 8$  and that would result being 48000 m/s.

**Very good, in the proper spirit of estimation, though units scandalously dropped:**

Tangential velocity and angular speed are related by the equation:

$$v_t = rw$$

Our  $r$  would be equal to 10,000 km (10,000,000 m), however, we still need to find the angular speed ( $w$ ) of the object. The formula for angular speed is:

$$w = d(\theta)/dt$$

Since we know that the Earth does one full rotation per day our value for  $d(\theta)$  would be  $2\pi$  radians. There are also about 86000 seconds in one day.

$$w = 2\pi/86000$$

$$w \sim 7 \times 10^{-5}$$

Now that we have found the angular speed ( $w$ ) we just need to multiply it by  $r$  to find the tangential velocity

$$v_t = (10,000,000)(7 \times 10^{-5})$$

$$v_t = 700 \text{ m/s}$$

**The angular velocity of the Earth orbiting around the Sun is approximately**

- a. 365 radians/day
- b. 0.017 radians/day
- c. 12 radians/yr
- d. 1.0 radians/yr