MA 231, Guided Notes §4.3

Recall:

Theorem: The First Derivative Test: Suppose that $x = c$ is a critical value for f (and $f(c)$ exists). Then:

1. If $f'(x)$ changes from positive to negative at $x = c$, then $(c, f(c))$ is a relative max (since f changes from increasing to decreasing there.)
2. If $f'(x)$ changes from negative to positive at $x = c$, then $(c, f(c))$ is a relative min (since f changes from decreasing to increasing there.)
3. If $f'(x)$ has the same sign on either side of $x = c$, then $(c, f(c))$ is neither a relative max nor a relative min.

Theorem: The Second Derivative Test for Relative Extrema: Suppose $x = c$ is a critical value for f and that $f''(c)$ is defined. Then

1. If $f''(c) < 0$, then $(c, f(c))$ is a relative max for f.
2. If $f''(c) > 0$, then $(c, f(c))$ is a relative min for f.

1 Optimization

We finally get to put derivatives to good use! The problems we study in this section are called ______________________________ . These are practical problems that require the use of calculus to find the desired solution, which will be either an absolute max or absolute min depending on the context.

Optimization problems can be challenging because they involve careful reading of the problem and translating it into mathematics. They are also all different, so there's no way to memorize what to do. We rely on a process, and pictures often help!

The optimization problem process:

1. Read the problem carefully – several times if necessary – until you understand the quantity that is to be ________________ .
2. With the aid of a large picture or diagram if appropriate, turn the information in the problem into equations.
   (a) The equation to be optimized is called the ________________ , or *target* equation.
   (b) Other equations are often called the *constraints*.
3. If necessary, use the constraint to turn the fundamental equation into an equation with just one variable.
4. Find the extreme values of the function using techniques from chapter 3.
5. State your solution in the context of the problem with appropriate units.

Example 1: Find two positive numbers whose sum is 60 and whose product is a maximum.

Example 2: Suppose we have 20 square feet of cardboard with which to make a box with square base and no top. Determine the dimensions of the box of maximum volume.

Example 3: We have \$1,000 to build a rectangular pen. Because of the terrain where the pen is to be built, the vertical sides (as viewed from above) cost \$5.00 per foot, and the other two sides cost \$7.00 per foot. Determine the dimensions of the pen that enclose the largest area.