EC-308: Homework 2

Due in class on Monday, October 24

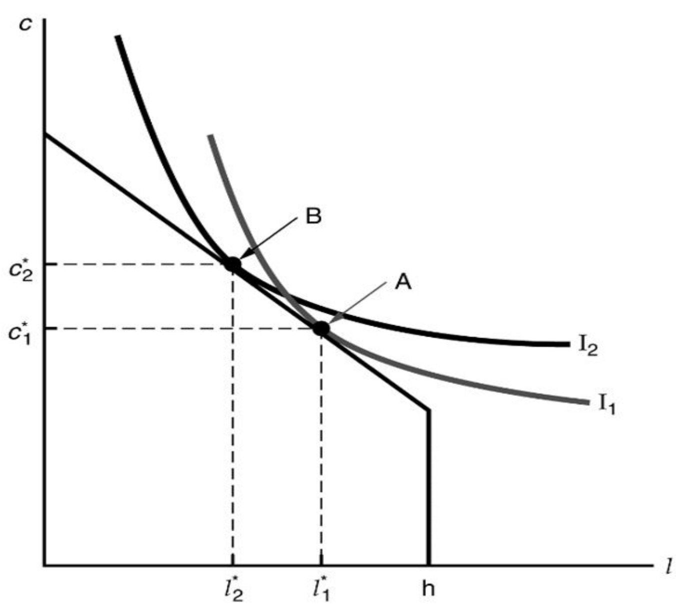
For full credit, please provide as detailed work as you can on the following problems. You can work in groups of 2 but must submit your own work. You should answer using graphs wherever possible.

Q1. Recall that leisure time in our model of the representative consumer is intended to capture any time spent not working in the market, including production at home such as yard work and caring for children. Suppose that the government were to provide free day care for children and, for the purpose of analyzing the effects of this, assume that this has no effect on the market real wage w, taxes T, and dividend income π. Determine the effects of the day care program on consumption, leisure, and hours worked for the consumer. Draw graphs to illustrate your answer. [Note: The primary objective of this question is to force you to think what happens when consumer's preferences over c and l change. Remember that leisure represents all time used for non-market activities.]

Solution:

The primary objective of this question is to force you to think what happens when consumer's preferences over c and l change. Remember that leisure represents all time used for non-market activities. Taking care of the children is also part of leisure (non-market activity because you as a parent don't get paid for it). If the government is now providing for some of those (government is paying you for childcare), like providing free child care, households will take advantage of such a program, thereby allowing more time for other activities, including market work. Concretely, this translates in a *change of preferences for households.* For the same amount of consumption, they are now willing to work more, or in other words, they are willing to forego some additional leisure. Remember that wage w doesn't change, so that cost of leisure is essentially the same as before. Only thing this government provided free childcare is doing is changing consumer's preference and thus incentivizing them to work more.

Figure 1: Government provided free childcare changes household preferences



On the figure above, the new indifference curve is labeled I_2. It can cross the original indifference curve I_1 because preferences, as we measure them here, have changed. The equilibrium basket of goods for the household now shifts from A to B. This

leads to reduced leisure (from l_1^* to l_2^*), and thus increased hours worked (higher labor supply), and increased consumption (from C_1^* to C_2^*) thanks to higher labor income(by working more hours) at the fixed wage. Note that we said two Indifference curves cannot cross *assuming the preferences are unchanged.* But if the preference do change (as in this question), they could cross, but then the indifference curves will be representing difference preferences, not the same preferences. In the diagram above, I_1 and I_2 represent different preferences.

Q2. In the course of producing its output, a firm causes pollution. The government passes a law that requires the firm to stop polluting, and the firm discovers that it can prevent the pollution by hiring 0.1 workers for every worker that is producing output. That is, if the firm hires N workers, then $0.1N$ workers are required to clean up the pollution caused by the N workers who are actually producing output. Determine the effect of the pollution regulation on the firm's profit-maximizing choice of labor input, and on the firm's labor demand curve. Show in graphs and explain.

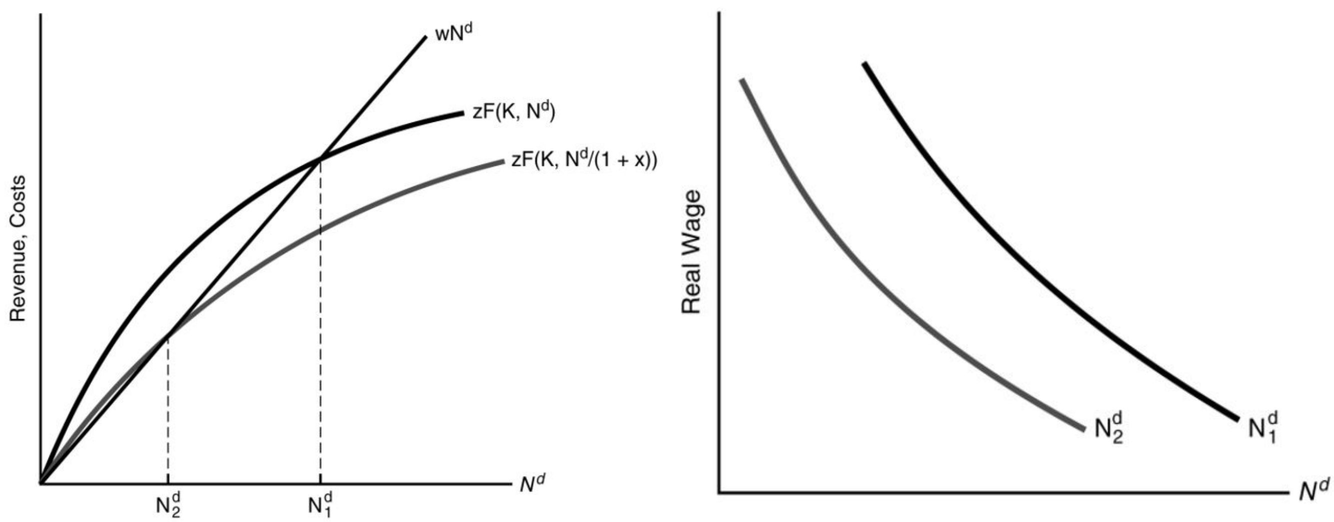
Solution:

As the firm has to internalize the pollution, it realizes that labor is less effective than it previously assumed. It now needs to hire $N(1+0.1) = 1.1N$ workers where N were previously sufficient. This is qualitatively equivalent to a reduction of z, total factor productivity.

Figure 2: Pollution regulation reduces firm's labor demand

(a) Firm's output declines

(b) Firm's labor demand declines



The figures above show these effects: the firm now hires fewer people for a given wage and thus its labor demand is reduced.

Q3. Suppose that the government subsidizes employment. That is, the government pays the firms t units of consumption goods for each unit of labor that the firm hires. Determine the effect of the subsidy on the firm's demand for labor. Show in graph and explain.

Solution:

The firm chooses its labor input N^d so as to maximize profits. When there is no subsidy, profits for the firm are given by

$$\pi = zF(K, N^d) - wN^d.$$

That is, profits are the difference between revenue and costs. In the first of the following set of figures, the the revenue function is $zF(K, N^d)$ and the cost function is the straight line, wN^d. The firm maximizes profits by choosing the quantity of labor where the slope of the revenue function equals the slope of the cost function, or where $MP_N = w$. The firm's demand for labor curve is the marginal product of labor schedule in the second of the following figures. With an employment subsidy, the firm's profits are:

$$\pi = zF(K, N^d) - (w - s)N^d,$$

where $zF(K, N^d)$ is the revenue (which is unchanged) and $(w - s)N^d$ is the new cost function. The subsidy acts to reduce the cost of each unit of labor by the amount of the subsidy, s. In the first of the following figures, the subsidy acts to shift down the cost function for the firm by reducing its slope. As before, the firm will maximize profits by choosing the quantity of labor input where the slope of the revenue function is equal to the slope of the cost function, $(t\breve{}s)$, so the firm chooses the quantity of labor where

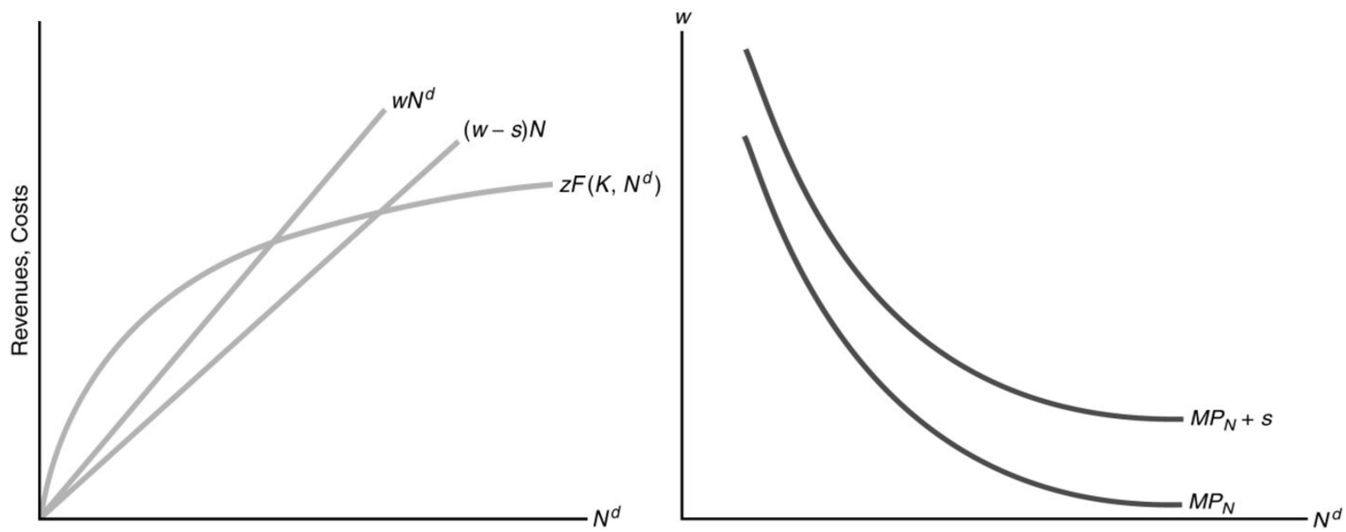
$$MP_N = w - s.$$

In the second figure, the labor demand curve is now $MP_N + s$, so that the labor demand curve has shifted up. The subsidy acts to reduce the marginal cost of labor, and the firm will hire more labor at any given real wage.

Figure 3: Effects of subsidizing employment

(a) Firm's profits increase

(b) Firm's labor demand increases



Q4. Suppose a firm called New England Patriots Inc. has a production function given by

$$Y = zK^{0.3}N^{0.7}.$$

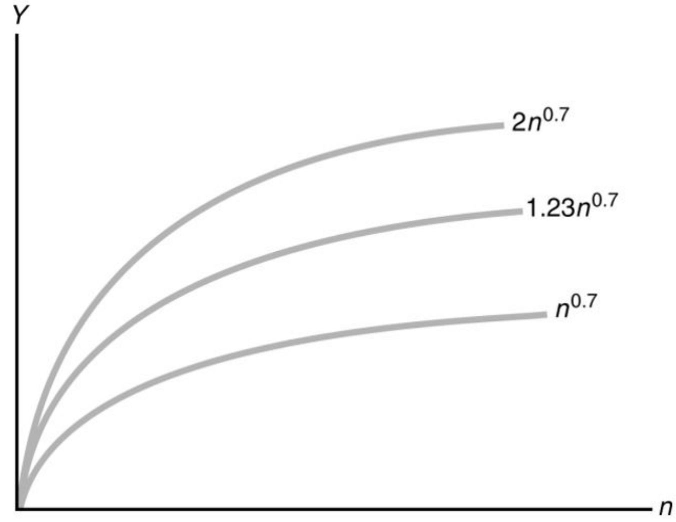
1. If $z = 1$ and $K = 1$, graph the production function. Is the marginal product of labor positive and diminishing? For graphing, you can consider values of $N = 5, 10, 15, 20, ...$

   Solution:

   The production function is $Y = 1 \times 1^{0.3} \times N^{0.7} = N^{0.7}$. The marginal product of labor is positive and diminishing, as shown in teh graph below.

Figure 4: $Y = N^{0.7}$

(a) The graph of $Y = n^{0.7}$ is labeled $n^{0.7}$(the bottom one)



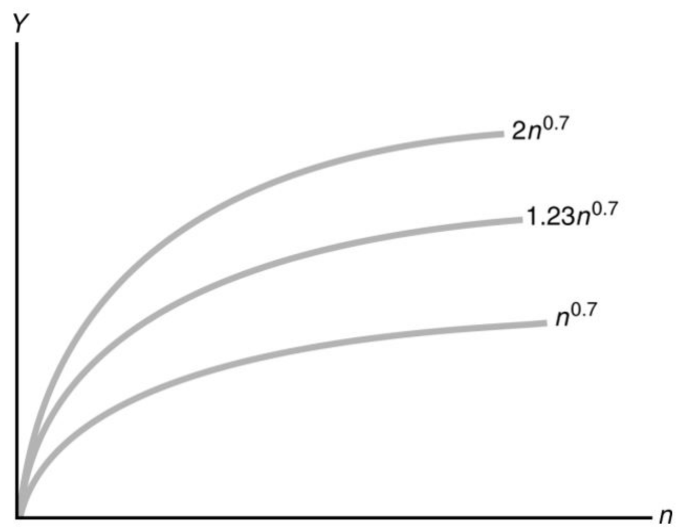
2. Now, graph the production function when $z = 2$ and $K = 1$. Explain how the production function changed from part 1.

Solution:

The production function is $Y = 2 \times 1^{0.3} \times N^{0.7} = 2N^{0.7}$. See the graph below.

Figure 5: $Y = 2N^{0.7}$

(a) The graph of $Y = 2N^{0.7}$ is labeled $2n^{0.7}$(the top one)



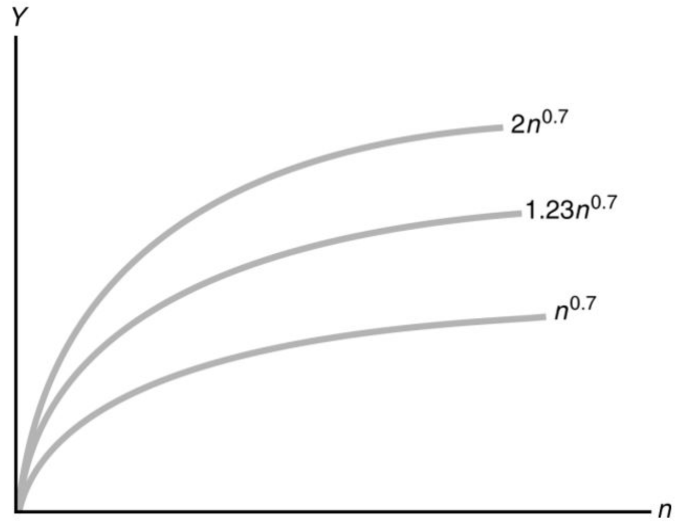
3. Next, graph the production function when $z = 1$ and $K = 2$. What happens now?

Solution:

The production function now becomes $Y = 1 \times 2^{0.3} \times N^{0.7} = 1.23N^{0.7}$. See the graph below.

Figure 6: $Y = 1.23N^{0.7}$

(a) The graph of $Y = 1.23N^{0.7}$ is labeled $1.23n^{0.7}$(the middle one)



4. Given the production function, graphs the MP_N for $(z, K) = (1, 1), (2, 1), (1, 2)$. Explain what you see.

Note: here, you must compute MP_N based on the production function given above. In order to do that you must compute $\frac{\partial}{\partial N}(zK^{0.3}N^{0.7})$. Note that ∂ means "partial" derivative. Here, you compute derivative of $zK^{0.3}N^{0.7}$ with respect to N, considering K a constant, like a number. One way to do that is to assume that $zK^{0.3} = b$, a constant. So we take our "regular" derivative $\frac{dbN^{0.7}}{dN} = \frac{bdN^{0.7}}{dN}$. Once this derivative is computed, we substitute the value of b in our answer and that's the *partial* derivative we want.

Solution:

First, we need to compute the marginal product of labor MP_N. It is the partial derivative of the production function with respect to labor N. Let $zK^{0.3} = b$ because this term is like a constant for our purpose. Then compute the derivative of $bN^{0.7}$ with respect to N.

$$\frac{dbN^{0.7}}{dN} = \frac{bdN^{0.7}}{dN} = 0.7N^{0.7-1} = 0.7bN^{-0.3}$$

Now simply substitute the value of b in $0.7bN^{-0.3}$ to obtain

$$MP_N = \frac{\partial}{\partial N}(zK^{0.3}N^{0.7}) = 0.7zK^{0.3}N^{-0.3}.$$

The graph looks like this

Figure 7: Graph of MP_N at given (z, K) combinations

(a) The bottom one is for $(z, K) = (1, 1)$, the middle one is for $(z, K) = (2, 1)$, and the top one is for $(z, K) = (1, 2)$