Worksheet 3: One-period model/ Static model

EC-308

October 17, 2022

0.0.1 A note on perfect complement and perfect substitute preferences

Perfect complement preferences

When consumer has perfect complement preferences, she consumes the consumption good and leisure in a fixed proportion. Mathematically, it looks like this:

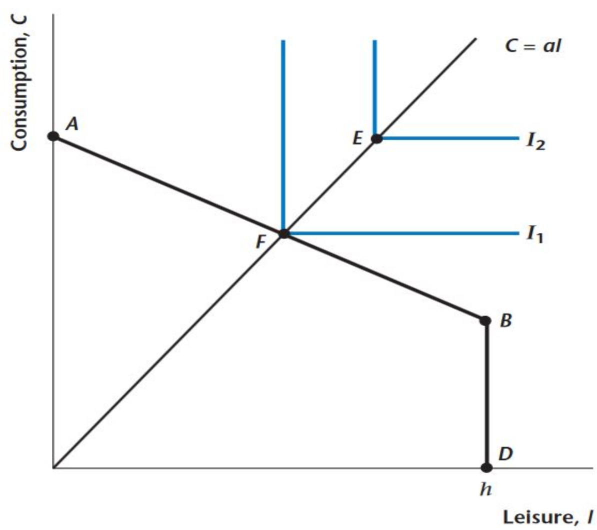
$$\frac{c}{l} = a.$$

This means that the ratio of c to l is constant (a). Multiplying both sides by l, we can re-write the above expression as

$$c = al.$$

When consumer has perfect complement preferences, Indifference curves are L shaped, as in the figure below.

Figure 1: Perfect complement preferences



The consumer's optimal bundle always lies on the line $c = al$, specifically at points F or E. Examples of perfect complement goods could be right shoe and left shoe, coffee and cream etc.

Perfect substitute preferences

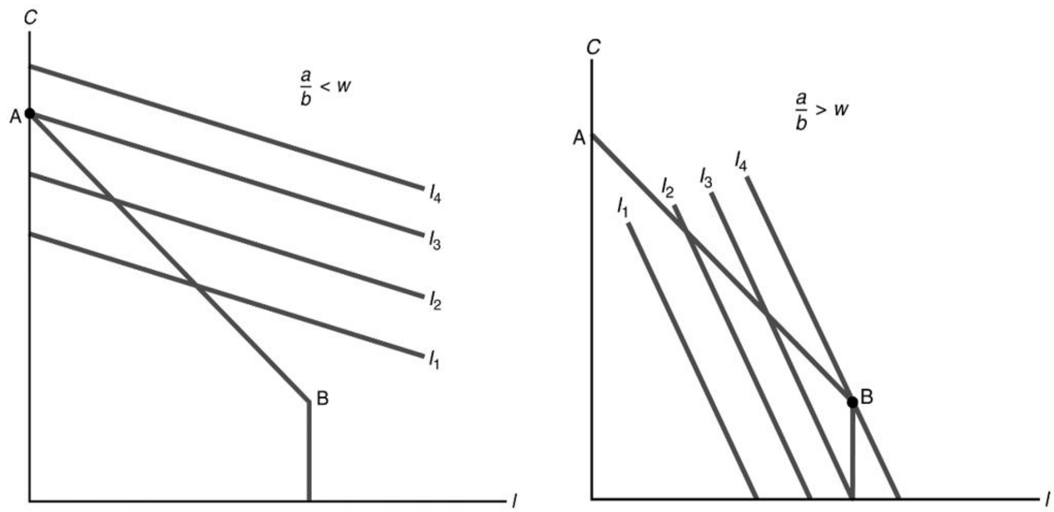
Perfect substitute preferences mean that consumer can consume leisure instead of consumption good or vice versa. While this example sounds strange, better examples of perfect substitute preferences are preference

over red pencil vs blue pencil, or, over brown rice vs pasta, United Airlines vs American Airlines etc. In this case, Indifference curves have this kind of expression:

$$u = al + bc,$$

where a and b are positive constants and u is the utility level. In graph, perfect substitute preferences look like this:

Figure 2: Perfect substitute preferences

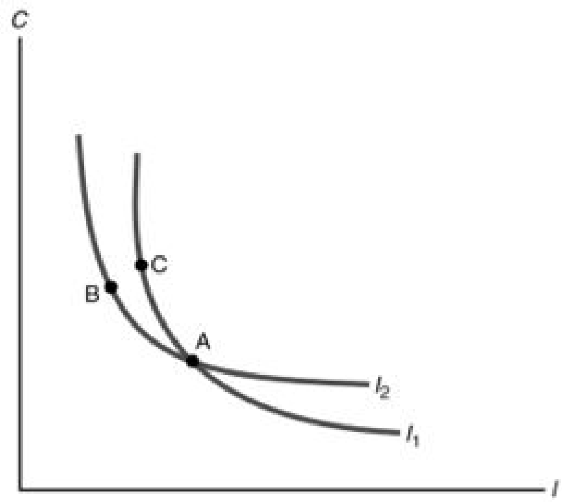


In the figure above, our representative consumer cannot optimize in either case because on the left graph, wage rate w is less than the $MRS_{l,c}$, which is $\frac{a}{b}$. On the graph on right, $MRS_{l,c}$ is higher than wage rate w. This means that the consumer optimizes when Indifference curve and the AB segment of the budget constraint overlap. Why? Because that's when we have $MRS_{l,c} = \frac{a}{b} = w$.

Note that in reality we don't quite see perfect complement or perfect substitutes. There are, however, many cases of gross complements or gross substitutes. The examples I gave above are are more of gross complements and gross substitutes.

0.0.2 Can indifference curves cross? Hint: remember what we assumed about consumer's preferences.

Figure 3: Can Indifference curves cross?



Answer:
Consider the two hypothetical indifference curves in the figure above. Point A is on both indifference curves, I_1 and I_2. By construction, the consumer is indifferent between A and B, as both points are on I_2. Similarly, the consumer is indifferent between A and C, as both points are on I_1. But at point C, the consumer has more consumption and more leisure than at point B. As long as the consumer prefers more to less, he or she must strictly prefer C to A. We therefore contradict the hypothesis that two indifference curves can cross.

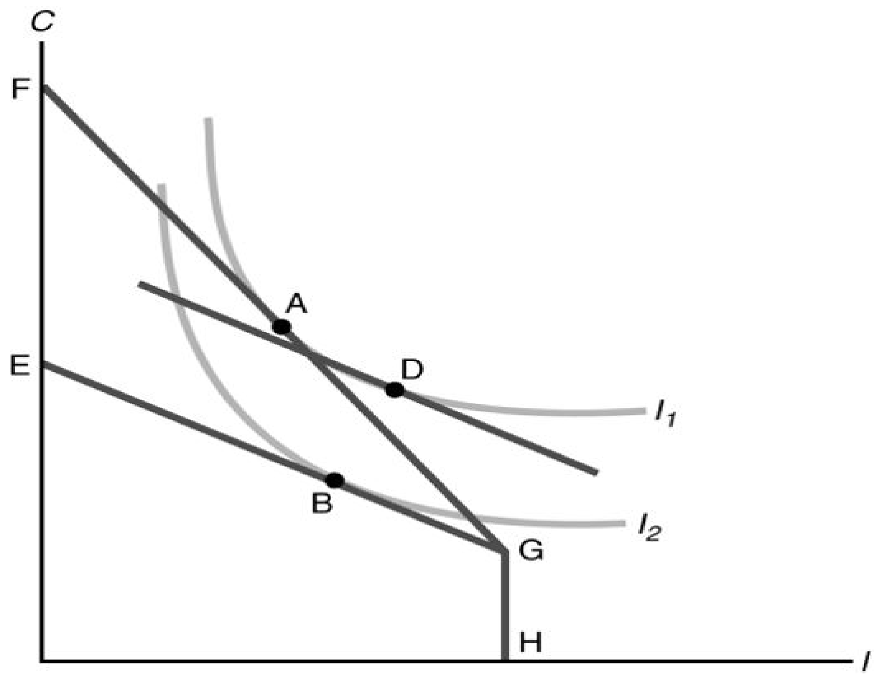
0.0.3 Suppose that the government imposes a proportional income tax on the representative consumer's wage income. That is, the consumers wage income is $w(1-t)(h-l)$ where t is the tax rate. What effect does the income tax have on consumption and labor supply? Explain your results in terms of income and substitution effects.

Answer:
With a proportional tax, budget constraint is

$$c = w(1-t)(h-l) - T,$$

where t is the tax rate on wage income.

Figure 4: Income and Substitution effect of proportional tax on wage income



where t is the tax rate on wage income. In the figure above, the budget constraint for $t = 0$ is FGH. When $t > 0$, the budget constraint is EGH. The slope of the original budget line is $-w$, while the slope of the new budget line is $-(1-t)w$. Initially the consumer picks the point A on the original budget line. After the tax has been imposed, the consumer picks point B. The substitution effect of the imposition of the tax is to move the consumer from point A to point D on the original indifference curve. The point D is at the tangent point of indifference curve, I_1, with a line segment that is parallel to EG. The pure substitution effect induces the consumer to reduce consumption and increase leisure (work less).

The tax also makes the consumer worse off, in that he or she can no longer be on indifference curve, I_1, but must move to the less preferred indifference curve, I_2. This pure income effect moves the consumer to point B, which has less consumption and less leisure than point D, because both consumption and leisure are normal goods. The net effect of the tax is to reduce consumption, but the direction of the net effect on leisure is ambiguous. The figure shows the case in which the substitution effect on leisure dominates the income effect. In this case, leisure increases and hours worked fall. Although consumption must fall, hours worked may rise, fall, or remain the same.

0.0.4 Suppose that a consumer cannot vary hours of work as he or she chooses. In particular, he or she must choose between working q hours and not working at all, where $q > 0$. Suppose that dividend income is zero, and that the consumer pays a tax T if he or she works, and receives unemployment insurance benefit b when not working.

1. If the wage rate increases, how does this affect the consumer's hours of work? What does this have to say about what we would observe about the behavior of actual consumers when wages change?
2. Suppose that the unemployment insurance benefit increases. How will this affect hours of work? Explain the implications of this for unemployment insurance programs.

Answer:

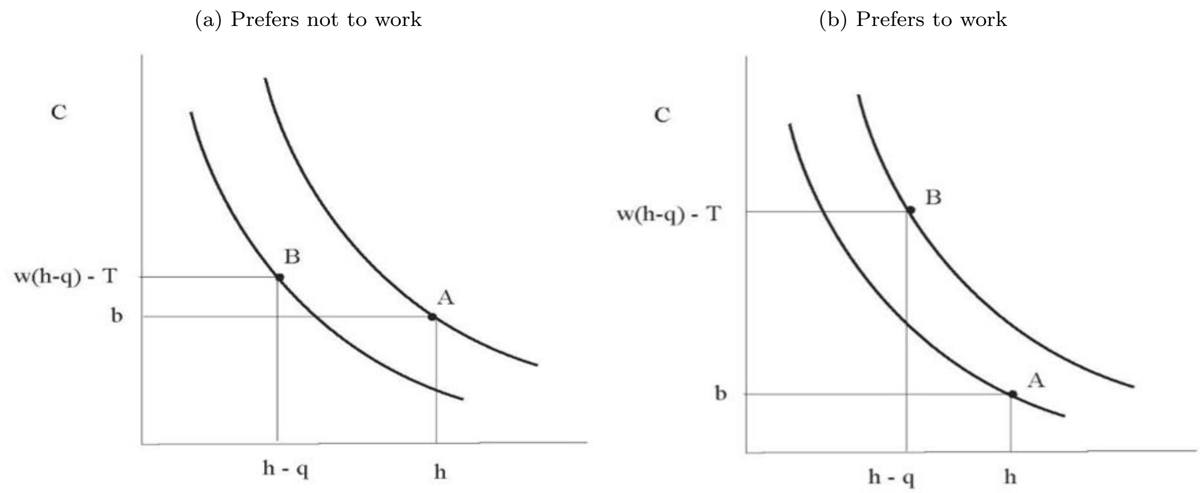
(1) Supposing that the only options open to the consumer are working q hours and paying a tax T, or

working zero hours and receiving an unemployment insurance benefit b, consumption c will be

$w(h-q)-T$ if the consumer works and,
b if the consumer decides not to work.

Then, either the consumer prefers not to work, as in the panel (a) in Figure 5 below, where the highest indifference curve is achieved at point A rather than at point B, or the consumer prefers to work, as in panel (b) in Figure 5. There is also another case where the consumer is just indifferent between working and not working, but that case is not important.

Figure 5: Consumer optimization



(2) Similar to part (1), if the unemployment insurance benefit increases, this will make not working preferable to some consumers who were formerly working, and employment will fall. An increase in the unemployment insurance benefit unequivocally reduces the quantity of labor supplied.

0.0.5 Suppose that a consumer can earn a higher wage rate for working overtime. That is, for the first q hours the consumer works, he or she receives a real wage rate of w_1, and for hours worked more than q he or she receives w_2,where $w_2 > w_1$.Suppose that the consumer pays no taxes and receives no nonwage income, and he or she is free to choose hours of work.

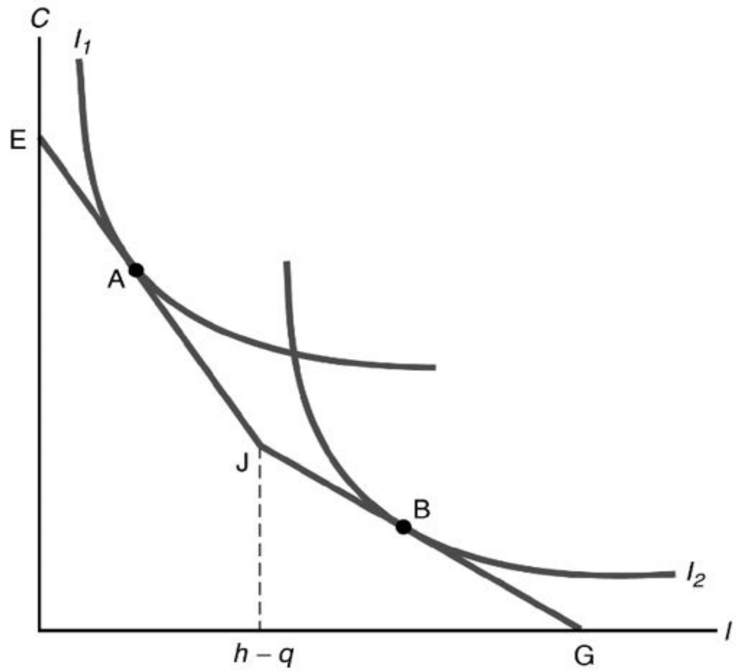
1. Draw the consumer's budget constraint,and show his or her optimal choice of consumption and leisure.
2. Show that the consumer would never work q hours, or anything very close to q hours.Explain the intuition behind this.
3. Determine what happens if the overtime wage rate w_2 increases. Explain your results in terms of income and substitution effects.You must consider the case of a worker who initially works overtime, and a worker who initially does not work overtime.

Answer:
This problem introduces a higher, overtime wage for hours worked above a threshold, q. This problem also abstracts from any dividend income and taxes.

(1) The budget constraint is now EJG in the figure below. The budget constraint is steeper for levels of leisure less than $h-q$, because of the higher overtime wage. The figure depicts possible choices for two

different consumers. Consumer 1 picks point A on her indifference curve, I_1. Consumer 2 picks point B on his indifference curve, I_2. Consumer 1 chooses to work overtime; consumer 2 does not.

Figure 6: Consumer optimization with overtime option



(2) The geometry of the figure above makes it clear that it would be very difficult to have an indifference curve tangent to EJG close to point J. In order for this to happen, an indifference curve would need to be close to right angled as in the case of pure complement. It is unlikely that consumers wish to consume goods and leisure in fixed proportions, and so points like A and B are more typical. For any other allowable shape for the indifference curve, it is impossible for point J to be chosen.

(3)An increase in the overtime wage steepens segment EJ of the budget constraint, but has no effect on the segment JG. For an individual like consumer 2, the increase in the overtime wage has no effect up until the point at which the increase is large enough to shift the individual to a point like point A. Consumer 2 receives no income effect because the income effect arises out of a higher wage rate on inframarginal units of work. An individual like consumer 1 has the traditional income and substitution effects of a wage increase. Consumer 1 increases her consumption, but may either increase or reduce hours of work according to whether the income effect outweighs the substitution effect.

0.0.6 Suppose that the government imposes a producer tax. That is, the firm pays t units of consumption goods to the government for each unit of output it produces. Determine the effect of this tax on the firm's demand for labor.

Answer:

The firm chooses its labor input, N^d, so as to maximize profits. When there is no tax, profits for the firm are given by

$$\pi = zF(K, N^d) - wN^d.$$

In the top figure on the following page, the revenue function is $zF(K, N^d)$ and the cost function is the straight line, wN^d. The firm maximizes profits by choosing the quantity of labor where the slope of the revenue function equals the slope of the cost function:

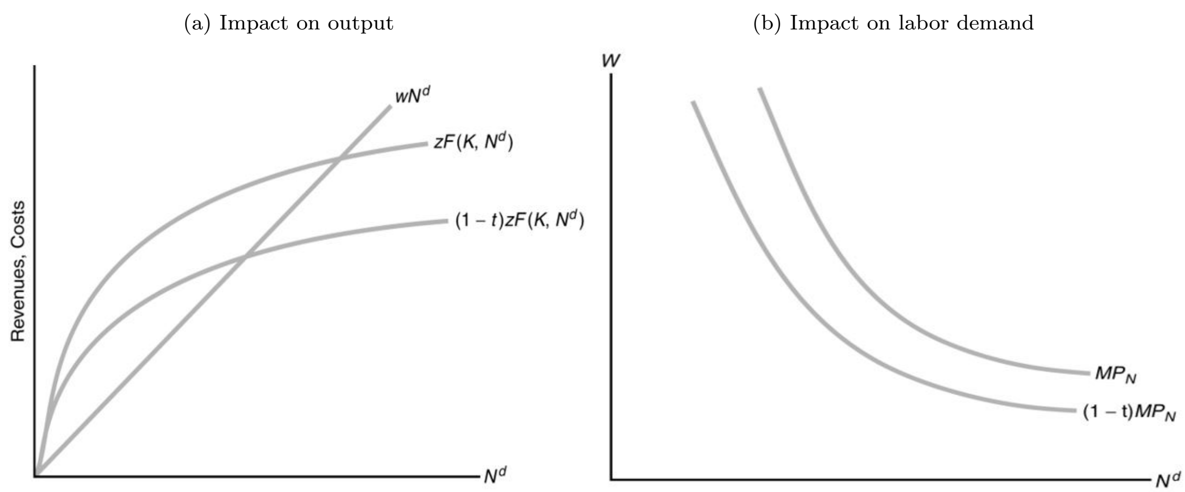
$$MP_N = w.$$

The firm's demand for labor curve is the marginal product of labor schedule in the panel(b).
With a tax that is proportional to the firm's output, the firm's profits are given by:

$$\pi = zF(K, N^d) - wN^d - tzF(K, N^d) = (1-t)zF(K, N^d).$$

Now, $(1-t)zF(K, N^d)$ is the after-tax revenue function and wN^d is the cost function.

Figure 7: Firm's optimization with a tax on output



In the above figures, the tax acts to shift down the revenue function for the firm and reduces the slope of the revenue function. As before, the firm will maximize profits by choosing the quantity of labor input where the slope of the revenue function is equal to the slope of the cost function, but the slope of the revenue function is

$$(1-t)MP_N,$$

so the firm chooses quantity of labor where

$$(1-t)MP_N = w.$$

In panel (b), the labor demand curve is now $(1-t)MP_N$,and the labor demand curve has shifted down. The tax acts to reduce the after-tax marginal product of labor, and the firm will hire less labor at any given real wage.