A ball on the end of a string is moving in a horizontal circle on a frictionless table. The other end of the string is threaded through a hole at the origin, and the string is slowly shortened by pulling it through the hole. Eventually, the radius of the circle is half of the original length. What happens to the angular speed of the ball? What happens to the linear speed? Explain your reasoning.

Key question: What remains constant? Is this rigid rotation?

When the radius of the circle is half of the original length, the angular speed remains the same. Although the circle becomes smaller, the angle associated with the ball remains the same. Linear speed, on the other hand, is different. When the radius is half of the original length, the linear speed of the ball is less. At original length, the ball travels a wider distance. Therefore, under these circumstances, the linear speed is greater when compared to that of the ball traveling around a circle with a smaller radius.

Is rotational energy constant?

In order to find the angular velocity, we will need to use the conservation of energy

$1/2(I_1)(w_1^2) = 1/2(I_2)(w_2^2)$

$1/2(mr^2)(w_1^2) = 1/2(m(r/2)^2)(w_2^2)$.

$w_2 = 2w_1$

With this solved we find that w_1 is multiplied by 2 therefore the angular velocity will be doubled when the radius of the circle is half its original length

Next, we will use the formula for linear speed

$v_1 = rw_1$

$v_2 = (r/2)w_2 = rw_1$

With the linear speed being solved, we see nothing has really changes therefor the linear velocity will be unchanged

Is it angular momentum?

As the radius of the circle is half of the original length then the angular speed of the ball increases as its moment of inertia decreases. Because isolated objects that maintains a constant value of angular momentum, change the moment of inertia which would change the angular velocity. The linear speed will increase because as the circles radius decreases half the original length than it will travel in a full revolution faster.