Angular Momentum

When you suddenly understand something, you can’t attribute that realization to the most recent input alone. It wouldn’t have done the trick without all the prior priming.

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her rotational inertia and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her final rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger because she’s rotating faster.
3. smaller because her rotational inertia is smaller.

ANS: **2**—Her kinetic energy increases as she pulls her arms in.

The forces pulling her arms inward are internal to the system, so they do not an external torque. Thus angular momentum is conserved. Recall that we can write translational kinetic energy in terms of momentum as $K_{\text{trans}} = P^2/2m$. By analogy, we can write rotational kinetic energy in terms of angular momentum: $K_{\text{trans}} = L^2/2I$. Because L is the same for both, her rotational kinetic energy will increase as her moment of inertia decreases.

This result makes sense. Her arms apply an inward force as she pulls them in. Therefore, she does positive work moving her arms inward. This additional work in the system becomes the initial rotational kinetic energy.

Motorcycle daredevil Awful Knawful attempts to jump a record 420 obsolete SUVs. While in midair, he revs the engine, making the back wheel spin faster. What happens to the motorcycle?

1. It moves faster through the air.
2. The front rises upward.
3. The front drops downward.
4. It turns to the left.
5. It turns to the right.
6. None of the above.

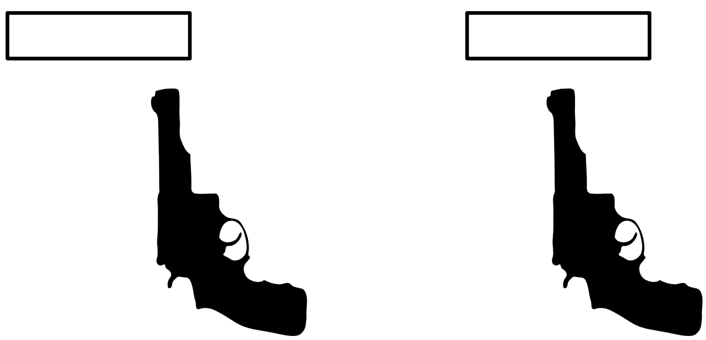
ANS: **2**—The front rises upward.

Ignoring air resistance, the only force on the motorcycle is gravity, which acts on the center of mass. Therefore, the angular momentum of the system around an axis through the center of mass must be conserved.

If you rev the motor, the rear (drive) wheel will speed up. Let’s say you’re viewing the motorcycle from its right side. The drive wheel will be spinning clockwise and speeding up. Therefore, the clockwise angular momentum will increase. To keep the total angular momentum constant, there must be an additional counter-clockwise angular momentum—the body of the motorcycle itself will rotate counterclockwise. Thus the front wheel will rise.

You can also think about this in terms of internal torques. In order to get the clockwise angular velocity of the drive wheel to speed up, there must be a clockwise torque on the wheel. This torque is exerted on the wheel by the motorcycle body (through the drive chain). Therefore, according to Newton’s third law (yes, there’s one for torques, too) the wheel must exert an equal and opposite counterclockwise torque on the motorcycle body, causing its front end to rise!

Two identical blocks of wood are placed above the barrels of identical pistols. These guns are both pointed upward, but one points at the center of a block, while the other points well off the center. Therefore, the block on the left will rotate significantly once the triggers are pulled. *The bullet is embedded in the block in both cases.*



Which block will rise to a higher maximum height?

1. Right block, with bullet in center
2. Left block, with bullet at one end
3. Same height
4. Need more information

ANS: **3**—The blocks will rise to the same maximum height.

There's a lot going on in this problem, so let's be careful. We know that the bullets will have equal momentum before collision, so the blocks (with embedded bullets) will have equal momentum after collision. This gives the blocks equal *translational* kinetic energy after collision ($K_{\text{trans}} = p^2/2m$). Therefore the blocks will rise to the same height as the translational kinetic energy is converted to gravitational potential energy. *Rotational considerations have nothing to do with this result!*

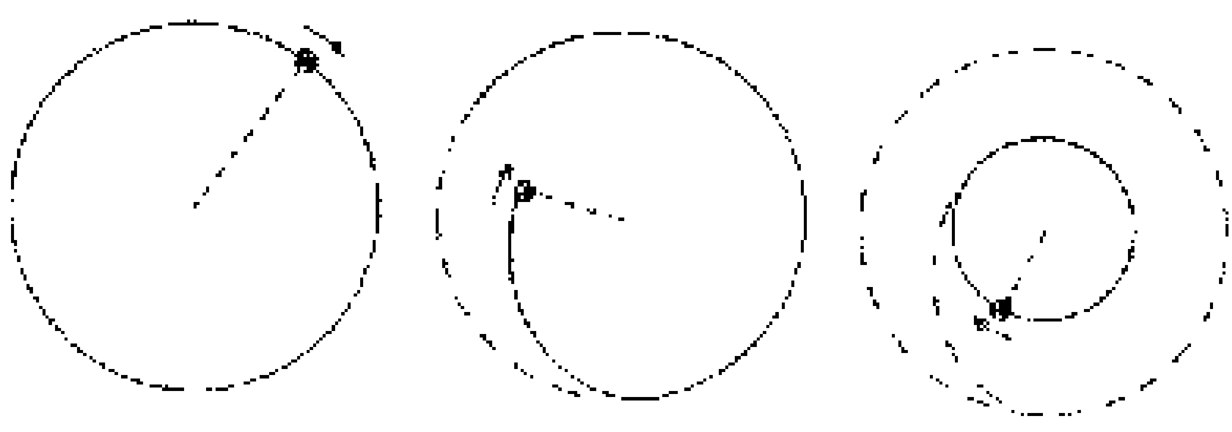
There is rotation in the problem, but it does not affect the answer. Angular momentum is conserved in the collision, so the block/bullet on the left will spin and have a good deal of rotational kinetic energy. The block/bullet on the right will have no rotational kinetic energy. However, because there are no dissipative torques as the blocks rise, their rotational kinetic energies will remain constant. It is only the translational kinetic energy that is converted to gravitational potential energy.

A common misconception is to think that the blocks should have the same mechanical energy after collision because the bullets had the same mechanical energy before. This is certainly not true because these collisions are *completely inelastic*. Mechanical energy is not conserved in the collision.

The block on the left clearly has more mechanical energy after the collision than does the block on the right. Why the difference? In both cases, mechanical energy will be lost in the collision. As the bullet bores into the wood, dissipative forces between the block and bullet take energy out of the system. Apparently the spinning block, which has more mechanical energy after collision, loses less energy in the inelastic collision. Therefore, we can predict with confidence that the bullet will be embedded more deeply into the block on the right, and less deeply in the spinning block on the left. (Less displacement for a given dissipative force means less negative work done.) If we cut the blocks open we will find this to be true.

Warmup Question

A ball on the end of a string is moving in a horizontal circle on a frictionless table. The other end of the string is threaded through a hole at the origin, and the string is slowly shortened by pulling it through the hole. Eventually, the radius of the circle is half of the original length. What happens to the angular speed of the ball? What happens to the linear speed? Explain your reasoning.



Warmup Question

Jupiter's moon Io ejects $1\,\text{ton}$ ($10^3\,\text{kg}$) of ionized sulfur and oxygen gas every second (wow!!!!). Even though Io is $400,000\,\text{km}$ from Jupiter, this gas is swept up into Jupiter's magnetic field and rotates with the planet at Io's orbit, making a complete circuit every ten hours. The entire cloud of gas encircles Jupiter like a huge donut in space, with a total mass of about a million tons. Estimate the total angular momentum of this cloud.

Warmup Question

A gymnast vaulting through the air (okay, through a vacuum!) can change her

1. Angular momentum
2. Rotational kinetic energy
3. Translational momentum
4. All of the above
5. None of the above