MA 231, Guided Notes §5.3

Recall: The definite integral of a function reports the total signed area between the function and the x-axis. The notation for the definite integral from $x = a$ to $x = b$ is

$$\int_a^b f(x)dx.$$

We did some graphical examples last time where we were either given the areas we needed or could find them by finding the areas of rectangles or triangles.

Today we tackle the much more difficult situation of needing to find the area under a curve where there isn't any formula.

The Big Idea:

1. Approximate the area we want with ________________ since we know how to find their areas.

2. For a better approximation, use more rectangles.

3. For the *exact* area, take a ___________ as the number of rectangles goes to infinity!

Example 1: Use 4 rectangles to approximate $\int_0^2 x^2 dx$.

Example 2: Use 8 rectangles to approximate $\int_0^2 x^2 dx$.

Example 3: Use an applet to find $\int_0^2 x^2 dx$.

__**Def:**__ **The Riemann Integral:** Given a function f that is continuous except for possibly at a finite number of points. Then we define the Riemann integral, $\int_a^b f(x)dx$ by the following process:

1. Partition the interval $[a, b]$ into n subintervals that will form the bases of the rectangles we'll use to approximate the area:

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b.$$

2. Note that the width of each subinterval (and base of each rectangle) is given by

$$\Delta x_i = x_i - x_{i-1},$$

for $i = 1, 2, \ldots, n$.

3. From each subinterval, select a "test point" $c_i \in [x_{i-1}, x_i]$, which will be the point at which we measure the heights of our rectangles.

4. Find the areas of the approximating rectangles by multiplying base times height: $f(c_i)\Delta x_i$.

5. Approximate the integral with the rectangles by adding them all up:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(c_i)\Delta x_i.$$

6. The exact value of the integral is now the limit as we let the number of rectangles go to infinity:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i.$$

Note: While it is possible to evaluate a Riemann integral using this definition, we'd like to avoid that when possible! Next time we'll see how.