MA 150 Notes §4.1

In this section we present one of the first mathematical models for the spread of an infectious disease: the **S-I-R model**, first developed by Kermack and McKendrick around 1927.

Questions an infectious disease model can help answer:

- How fast will the disease spread?
- How many will become infected?
- How many people need to be vaccinated to stop the epidemic?
- How should resources be allocated between prevention and treatment?

The S-I-R model splits a population into three categories of interest (so these will all get ovals in our flow diagram!):

- $S(t)$ = the number of ________________ people after t days. (These are people who can get the disease.)
- $I(t)$ = the number of ________________ people after t days. (These are people who have the disease *and* can spread it.)
- $R(t)$ = the number of ________________ people after t days. (These are people who have had the disease and recovered with immunity or dies.)

Flow Diagram Start: Draw all possible arrows among the three ovals and explain what they would mean.

Model parameters:

- $\beta =$ the *effective contact rate*. This is the average number of ______________________________ contacts an infective person has on average each day.
- $\delta =$ the *duration of infectivity*. This is the average length of time someone with the disease ______________________________ to others.
- $N =$ the total population, so $S(t) + I(t) + R(t) = N$.

Flow Diagram:

DDS:

$$
\begin{aligned}
S(t) &= S(t-1) - \beta I(t-1) \cdot \frac{S(t-1)}{N} \\
I(t) &= I(t-1) + \beta I(t-1) \cdot \frac{S(t-1)}{N} - \frac{1}{\delta} I(t-1) \\
R(t) &= R(t-1) + \frac{1}{\delta} I(t-1)
\end{aligned}
$$

Excel!

Class: Sketch a prediction of what the graph of each variable will look like.

Eyam Plague:

Numbers of Susceptible, Infective, and Removed During the Eyam Plague			
Date 1666	Susceptibles	Infectives	Removed
June 19	254	7	0
July 3	235	15	11
July 19	201	22	38
August 3	154	29	78
August 19	121	20	120
September 3	108	8	145
September 19	97	8	156
October 4	Incomplete data	Incomplete data	167
October 20	83	0	178