MA 150 Notes §4.1b

Recall: The basic S-I-R model is given by

$$\begin{aligned} S(t) &= S(t-1) - \beta I(t-1) \cdot \frac{S(t-1)}{N} \\ I(t) &= I(t-1) + \beta I(t-1) \cdot \frac{S(t-1)}{N} - \frac{1}{\delta} I(t-1) \\ R(t) &= R(t-1) + \frac{1}{\delta} I(t-1), \end{aligned}$$

where

- $S(t) =$ the number of susceptible people after t days;
- $I(t) =$ the number of infective people after t days; and
- $R(t) =$ the number of removed people after t days.

Model parameters:

- $\beta =$ the *effective contact rate*.
- $\delta =$ the *duration of infectivity*.
- $N =$ the total population, so $S(t) + I(t) + R(t) = N$.

The Basic Reproductive Rate: R_0, or "R-naught," is the notation for the *basic reproductive rate* for a disease. It is one of the first numbers an epidemiologist will want to know about a disease because it provides a measure for

____ .

Def: R_0 = the number of infections a single infective person would cause if everyone else in the population was ____________________ . For the basic S-I-R model,

$$R_0 = \beta \times \delta.$$

Example:

Threshold Theorem I: An epidemic begins to die out when the

______________________________ drops below

$$\frac{1}{R_0}.$$

Equivalently,

Threshold Theorem II: An epidemic begins to die out when the

______________________________ drops below

$$\frac{N}{R_0}.$$

The reason the threshold theorems are true comes from thinking carefully about the infective category:

Example:

Herd Immunity and Vaccinations: The threshold theorems tell us how many people we need to vaccinate to prevent an epidemic!

Def: *Herd immunity* is the phenomenon where we can prevent an epidemic by vaccinating ______________________________ of the population.

Herd Immunity Theorem: In order to prevent an epidemic, the proportion of the population we must vaccinate is

$$1 - \frac{1}{R_0}.$$

Note that the larger the R_0, the more people we need to vaccinate, i.e. the harder the disease is to control!

Example: