MA 150 Notes §4.2

Recall: The basic S-I-R model is given by

$$\begin{aligned}
S(t) &= S(t-1) - \beta I(t-1) \cdot \frac{S(t-1)}{N} \\
I(t) &= I(t-1) + \beta I(t-1) \cdot \frac{S(t-1)}{N} - \frac{1}{\delta} I(t-1) \\
R(t) &= R(t-1) + \frac{1}{\delta} I(t-1),
\end{aligned}$$

where

- $S(t)$ = the number of susceptible people after t days;
- $I(t)$ = the number of infective people after t days; and
- $R(t)$ = the number of removed people after t days.

Model parameters:

- β = the *effective contact rate.*
- δ = the *duration of infectivity.*
- N = the total population, so $S(t) + I(t) + R(t) = N$.

The Basic Reproductive Rate: R_0 is the number of infections a single infective person would cause if everyone else in the population was susceptible. For the basic S-I-R model,

$$R_0 = \beta \times \delta.$$

Vital Dynamics *Vital dynamics* means changes in the population such as new births and deaths due to causes besides the disease. In this section we include vital dynamics in our model, but we do so in a way that still keeps the total population ________________ .

- We let μ (lower case Greek "mu") be the daily ______________________ for the population. This means that every day there are ___________ new births added to the population.

- To keep the population constant, we have to let μ be the daily ______________________ (for deaths not caused by the disease).

- We assume that non-disease deaths affect all categories equally, so each day we lose μS, μI, and μR from each category.

- Note that $\mu S + \mu I + \mu R =$ ___________!

Flow Diagram:

DDS:

Excel!

The Waiting Time Principle: For any compartment in a DDS, the average amount of time someone spends in the compartment equals the reciprocal of the proportion who leave each day (if constant).

$$\text{Days Spent in Compartment} = \frac{1}{\text{Proportion Leaving Each Day}},$$

or

$$\text{Proportion Leaving Each Day} = \frac{1}{\text{Days Spent in Compartment}}.$$

Basic Reproductive Rate, R_0: The inclusion of vital dynamics changes the formula for R_0, because people on average do not spend the full average duration in the infective category – some will die before then.

$$R_0 = \frac{\beta \cdot \delta}{1 + \mu \cdot \delta}.$$

Why is this true?

Example:

Threshold Theorem I: An epidemic wanes when the proportion of susceptibles is below

$$\frac{1}{R_0}.$$

Threshold Theorem II: An epidemic wanes when the number of susceptibles is below

$$\frac{N}{R_0}.$$

Herd Immunity Theorem: In order to sustain protection against an epidemic, the proportion of newborns that must be vaccinated must be at least

$$1 - \frac{1}{R_0}.$$