MA 150 Notes §4.3

Recall: The basic S-I-R model with vital dynamics is given by

$$\begin{aligned}
S(t) &= S(t-1) - \beta I(t-1)\cdot\frac{S(t-1)}{N} + \mu N - \mu S(t-1)\\
I(t) &= I(t-1) + \beta I(t-1)\cdot\frac{S(t-1)}{N} - \frac{1}{\delta}I(t-1) - \mu I(t-1)\\
R(t) &= R(t-1) + \frac{1}{\delta}I(t-1) - \mu R(t-1),
\end{aligned}$$

where

- $S(t)$ = the number of susceptible people after t days;
- $I(t)$ = the number of infective people after t days; and
- $R(t)$ = the number of removed people after t days.

Model parameters:

- β = the *effective contact rate*, δ = the *duration of infectivity.*
- μ = the daily birth/death rate, N = the total population, so $S(t) + I(t) + R(t) = N$.

The Basic Reproductive Rate: R_0 is the number of infections a single infective person would cause if everyone else in the population was susceptible. With vital dynamics,

$$R_0 = \frac{\beta\delta}{1+\mu\delta}.$$

Using Data to Find R_0:

For endemic diseases:

By Algebra in Text (check with Excel): With vital dynamics, the long-term number of susceptibles is given by the equilibrium value:

$$S^* = \underline{\hspace{3cm}}.$$

Then

$$\frac{S^*}{N} = \underline{\hspace{3cm}}.$$

The fraction of people who are susceptible is something we can measure with blood tests! Use a **serosurvey** to test for antibodies to estimate the fraction susceptible.

$$\frac{S^*}{N} \approx \frac{\text{Number without antibodies}}{\text{Total tested}} \approx \frac{1}{R_0}.$$

So,

$$R_0 \approx \frac{1}{\text{Proportion without antibodies}}.$$

Example: Rubella in The Gambia. Serosurveys showed 94% of the population had antibodies for rubella, thus

$$R_0 \approx \underline{\hspace{3cm}}.$$