

MA 150 Notes §4.4

Recall: *The Waiting Time Principle:* For any compartment in a DDS, the average amount of time someone spends in the compartment equals the reciprocal of the proportion who leave each day (if constant).

$$\text{Days Spent in Compartment} = \frac{1}{\text{Proportion Leaving Each Day}},$$

or

$$\text{Proportion Leaving Each Day} = \frac{1}{\text{Days Spent in Compartment}}.$$

Life Expectancy: If μ is the daily death (and birth) rate, then by the Waiting Time Principle, we have:

$$\text{Days Spent Alive} = \frac{1}{\text{Proportion Dying Each Day}},$$

or

or

$$L = \frac{1}{\mu},$$

where L = average life expectancy in days.

Average Age at Infection The average age at infection is the average age at which someone contracts the disease. As far as our model is concerned, the average age at infection is just the average amount of time someone spends in the susceptible category. Again we can apply the Waiting Time Principle to see how to find it:

$$\text{Days Spent in Susceptible Category} = \frac{1}{\text{Proportion Leaving S Each Day}},$$

or

Though we refer to the text for the algebraic details, the average age at infection, A , is affected by life expectancy, R_0 , and the vaccination proportion, ρ , where we assume ongoing vaccinations of newborns. The formula for average age at infection is

$$A = \frac{L}{R_0(1 - \rho)}.$$

Example: Find the average age at infection for a disease where $R_0 = 10$ in a country where $L = 75$, and 60% of newborns are vaccinated. What if no one is vaccinated?

The next example shows why understanding the relationship between vaccinations and average age at infection is so important.

Example: *Congenital Rubella Syndrome, CRS.* Rubella is a disease that is usually mild, with one important exception. If contracted by a pregnant woman, the result can be CRS, and the consequences for the baby can be profound, including blindness, developmental problems, and heart defects. Here we examine the question of whether or not to vaccinate for rubella in The Gambia.