

MA 150 Notes §8.2

Recall:

- The constant calorie expenditure model (CCEM) is given by

$$W(t) = W(t-1) + \frac{I_0}{3500} - \frac{E_0}{3500}.$$

- I_0 = the number of calories consumed each day, and E_0 = the number of calories burned each day.
- We assume that there are 3500 calories in a pound of body weight.
- The CCEM is not a good model for longterm projections because a person's weight will eventually go to zero or increase without bound.

1 Variable Calorie Expenditure Model

The problem with the CCEM is that it assumes that a person will burn the same number of calories per day no matter how much they weigh. In reality, our _____ change as our body weight changes. What we need is a better way to estimate how many calories a person will burn in a day.

1.1 Resting Energy Expenditure (REE)

A person's *resting energy expenditure* is the number of calories a person would burn in a day if they were completely at rest all day.

The Mifflin-St. Jeor Equation: The Mifflin-St. Jeor equation is one of many equations available for estimating a person's REE. Crucially, it depends on _____.

$$REE = 4.536 \cdot W + 15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161.$$

- $W =$ _____.
- $H =$ _____.
- $A =$ _____.
- $S =$ _____.

Example 1: Calculate Dr. Barton's REE using the Mifflin-St. Jeor equation.

1.2 Activity Level

In order to get an estimate for how many calories a person actually burns in a day, we multiply the REE by a factor known as the *activity level*, which we denote by λ_0 . The table below provides guidance for how to select λ_0 :

Activity Level	Description	λ_0
Sedentary	Desk job, little physical activity	1.2
Light Activity	Jobs involving some standing such as retail sales, some walking as exercise, light housework	1.375
Moderate Activity	Mason, construction worker, or sedentary occupation with daily hour of moderate intensity exercise	1.55
High Activity	Strenuous work or exercise for several hours daily, hard manual labor such as non-mechanized farming, or non-sedentary occupation with 2 hours of moderate to intense exercise daily	1.725
Extreme Activity	Multiple bouts of long and intense exercise daily such as for serious athletes in season, or a strenuous occupation with additional leisure exercise	1.9

We can now write down an expression for our daily calorie expenditure by multiplying our activity level times our REE. Note that because of the Mifflin-St. Jeor equation, our expenditure now depends on our body weight so it changes over time!

$$E(t-1) = \lambda_0 \cdot [4.536 \cdot W(t-1) + 15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161] .$$

1.3 Putting It All Together

First we start with the CCEM but with E_0 replaced by $E(t-1)$ to indicate that expenditure is changing:

$$W(t) = W(t-1) - \frac{E(t-1)}{3500} + \frac{I_0}{3500} .$$

Next we substitute for $E(t - 1)$:

$$W(t) = W(t - 1) - \frac{\lambda_0}{3500} \cdot [4.536 \cdot W(t - 1) + 15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161] + \frac{I_0}{3500}.$$

Finally, we collect all of the constant terms at the end of the equation to get our

Variable Calorie Expenditure Model:

$$W(t) = W(t - 1) - \frac{\lambda_0 \cdot 4.536}{3500} \cdot W(t - 1) + \frac{I_0}{3500} - \frac{\lambda_0}{3500} \cdot [15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161].$$

This model looks complicated, but it's really just another affine model

$$W(t) = W(t - 1) + rW(t - 1) + a,$$

$$\text{where } r = -\frac{\lambda_0 \cdot 4.536}{3500}, \text{ and } a = \frac{I_0}{3500} - \frac{\lambda_0}{3500} \cdot [15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161].$$

Since the model is an affine model, we know how to find the equilibrium value:

$$W^* = -\frac{a}{r} = \frac{\frac{I_0}{3500} - \frac{\lambda_0}{3500} \cdot [15.875 \cdot H - 5 \cdot A + 166 \cdot S - 161]}{\frac{\lambda_0 \cdot 4.536}{3500}}$$

After a bit of simplifying, we get:

$$W^* = 0.2205 \cdot \frac{I_0}{\lambda_0} - 3.5 \cdot H + 1.102 \cdot A - 36.596 \cdot S + 35.494.$$

Example 2: Determine Dr. Barton's longterm weight if his activity level is $\lambda_0 = 1.55$ and he consumes 2600 calories per day. Confirm your result using the Variable Calorie Expenditure Model provided on Moodle.