MA 207 Exam 2 Review **Name:** Key

1. Applications for a driver's license (which includes an exam) in a certain city are either satisfactory (S) or declined (D). Applicants are also asked to report on whether they completed driver's education training—yes (Y) or no (N). The contingency table below summarizes the data. Give answers as fractions and as decimals to three decimal places.

	Satisfactory (S)	Declined (D)	Total
Completed training (Y)	59	5	64
Did not complete (N)	61	7	68
Total	120	12	132

a. Find P(S).
$\frac{120}{132} = .909$

b. Find P(S|Y).
$\frac{59}{64} = .922$

c. IS P(S) = P(S|Y): Yes or no? No explanation is needed for this part.
No!

d. Complete the definition: S and Y are independent events if...
P(S) = P(S|Y).

e. Are S and Y independent events: Yes or no?
No!

f. Find P(Y).
$\frac{64}{132} = .485$

g. Using the table (rather than a probability rule), find P(Y and S).
$\frac{59}{132} = .447$

h. Find P(Y)P(S). (Hint: just multiply two fractions you have already found. Do <u>not</u> round intermediate calculations.)
$\left(\frac{64}{132}\right)\left(\frac{120}{132}\right) = .441$

i. Is P(Y and S) = P(Y)P(S): Yes or no? Why doesn't this violate the Multiplication Rule for Independent Events?
No! Because they are not independent events.

j. State the Multiplication Rule (that always works, not the one that only works for independent events).
P(Y and S) = P(Y)P(S|Y)

k. Using the multiplication rule, find P(Y and S). Did you get the same answer as part (g)? Yes or no?
P(Y and S) = P(Y)P(S|Y) = $\left(\frac{64}{132}\right)\left(\frac{59}{64}\right) = \frac{59}{132} = .447$
Yes!

l. Find P(Y|S).
$\frac{59}{120} = .492$

m. Is P(S|Y) = P(Y|S): Yes or no? Would you expect them to be equal?
No and no!

n. Complete the following definition: S and Y are disjoint events if...
they have no outcomes in common.

o. Are S and Y disjoint events? Why or why not?
No! They have outcomes in common (59 of them, in fact!)!

p. Find P(Y or S).
$$\frac{64}{132}+\frac{120}{132}-\frac{59}{132}=\frac{125}{132}=.947$$

q. Are S and D disjoint events? Why or why not?
Yes! They have no outcomes in common.

r. Find P(S or D).
$$\frac{120}{132}+\frac{12}{132}-0=\frac{132}{132}=1$$

s. Find $P(Y^C)$.
$$1-\frac{64}{132}=\frac{68}{132}=.515$$

2. You have a standard deck of 52 cards (no jokers). The deck has 4 suits in all—two red suits (hearts and diamonds) and two black suits (spades and clubs). Each suit has 13 ranks (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king).

a. Find the probability that you draw one card and it is a heart.
13/52 = 1/4

b. Find the conditional probability that you draw one card and it is a heart, given that it is a red card.
13/26 = 1/2

c. Are the events of drawing a heart and drawing a red card independent? Yes or no? No
d. Are the events of drawing a heart and drawing a red card disjoint? Yes or no? No
e. Are the events of drawing a queen and drawing an ace independent? Yes or no? No
f. Are the events of drawing a queen and drawing an ace disjoint? Yes or no? Yes
g. Are the events of drawing a heart and drawing an ace independent? Yes or no? Yes
h. Are the events of drawing a heart and drawing an ace disjoint? Yes or no? No
i. Is it possible for two events two be both disjoint and independent? Yes or no? No

j. Find the probability that you draw one card and it is a heart and an ace.
$$P(\text{heart})P(\text{ace}\mid\text{heart})=\left(\frac{13}{52}\right)\left(\frac{1}{13}\right)=\frac{1}{52}$$

k. Find the probability that you draw one card and it is a heart or an ace.
$$\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}$$

l. Find the probability that you draw three cards without replacement and none of them are hearts.
$$\left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) \text{ or } 0.4135$$

m. Find the probability that you draw three cards without replacement and at least one is a heart.
$$1-\left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) \text{ or } 0.5865$$

3. The price of gas, X, was recorded on a particular day at a random sample of gas stations and found to be approximately normal with a mean of \$2.50/gal and a standard deviation of \$0.25/gal. Include units in your answers whenever appropriate.

   a. What percent of gas stations charged less than 3 dollars per gallon? Make and label a drawing and shade an appropriate region.

      $z = (3\text{-}2.50)/.25 = 2$

      $P(X < 3) = P(z < 2)$ = .975 using the empirical rule or
      = .9772 using the standard normal table.

   b. What percent of gas stations charged less than \$2.40/gal? Make and label a drawing and shade an appropriate region. Give your answer as a decimal to four decimal places.

      $z = (2.40\text{-}2.50)/.25 = -.4$

      $P(X < 2.40) = P(z < -.4) = .3446$ using the standard normal table.

   c. What percent of gas stations charged between \$2.40/gal and \$3.00/gal? Make and label a drawing and shade an appropriate region. Give your answer as a decimal to four decimal places.

      $P(2.40 < X < 3.00) = P(X < 3.00) - P(X < 2.40)$ = .975 - .3466 = .6304 if you used the rule in part (a)
      = .9772 - .3466 = .6326 if you used the table in part (a)

   d. What was the price of gas at a station with z-score of 3? Include units.

      $3 = (X - 2.50)/.25$
      $3(.25) = X - 2.50$
      $X = 2.50 + 3(.25) = 3.25$ dollars per gallon

   e. What is the probability that a station had a price cheaper than 3 standard deviations below the mean? Give your answer to 4 decimal places.

      $P(z < -3)$ = .0015 using the empirical rule or
      = .0013 using the standard normal table.

   f. How much did gas cost at a station in the 3rd percentile? Make and label a drawing and shade an appropriate region. Interpret your answer (explain what your answer means in the context of the original problem using a complete sentence with correct grammar and punctuation that includes units).

      Look in the *body* of the standard normal table for the left tail probability closest to 0.03.
      The corresponding z-value is $z = -1.88$

      Then find the corresponding X-value:
      $-1.88 = (X - 2.50)/.25$ $\quad -1.88(.25) = X - 2.50$ $\quad X = 2.50 - 1.88(.25) = 2.03$ dollars per gallon.

      Any gas station in the 3rd percentile would have charged \$2.03 per gallon on that day. This means that approximately 3% of all gas stations (in the vicinity of the random sample) charged \$2.03 per gallon or less.

4. A student earned the scores listed below on various course components that make up the stated percentage of the grade. Find the course grade, which is the mean (weighted average) of the scores (also called the expected value).

Course Component	Percent of Course Grade	Student's Score
Exams	60%	82
Homework	15%	98
Project	5%	85
Final Exam	20%	79

a. Find the course grade, which is the mean (weighted average) of the scores (also called the expected value).

$\mu = 82(.60) + 98(.15) + 85(.05) + 79(.20) = 83.95$

b. Find the grade the student would have needed to earn an B+ (87%).

Solve $\mu = 82(.60) + 98(.15) + 85(.05) + x(.20) = 87$ for x to get $x = 94.25$

Ex 5 In Pick 3, you pay \$0.50 to play. You then choose 3 digits (order matters). If you correctly choose the 3 digits that are drawn you win \$275. What is your net expected value? If you play 40 times, how much money can you expect to gain/lose? If you play 4000 times?

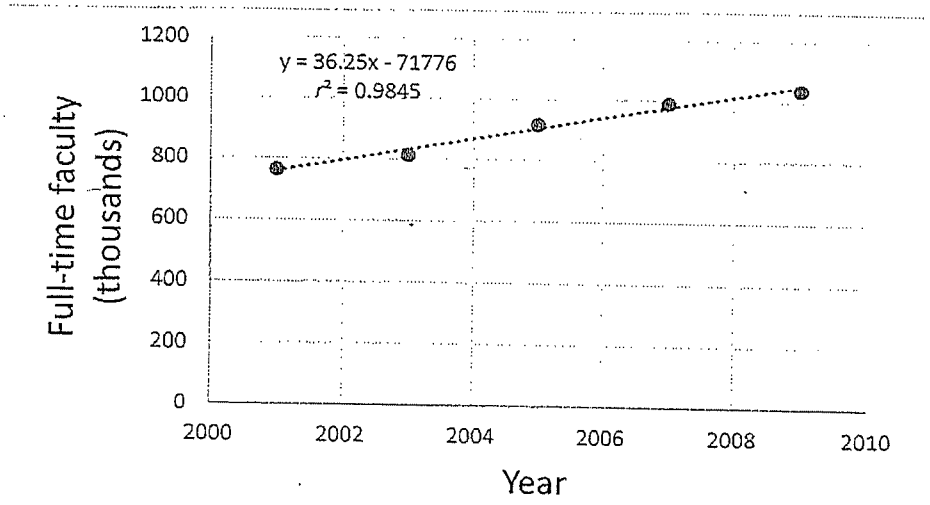
x	$P(x)$	$xP(x)$
274.50	$\frac{1}{1000}$	.2745
−.50	$\frac{999}{1000}$	−.4995

$\mu = \sum x_i P(x_i) = -.225$

If you play 40 times:
$40\mu = 40(-.225) = -\$9.$

If you play 4000 times:
$4000\mu = 4000(-.225) = -\$900.$

4. The scatter plot below shows number of full-time faculty (in thousands) at four year colleges in certain years, together with the equation of the least squares regression line and the value of r^2.



(a) What is the explanatory variable? Year

(b) What is the response variable? Faculty

(c) Describe the overall pattern (form, direction, strength) and any striking deviations from the overall pattern (regression outliers).

Linear, positive, strong, No outliers.

(d) In a complete sentence, explain what the value of r^2 means in the context of this problem. Be sure to include the actual value of r^2 in your explanation.

98.45% of the variation in the number of full-time faculty can be explained by the year.

(e) Does the passage of time (an increase in the year) cause an increase in the number of faculty? NO!!!

(f) What does the regression line predict for 2007? Give your answer to two decimal places and include units. Show your work using the equation of the regression line. Is this extrapolation? Yes or No?

$y = 36.25(2007) - 71776 =$ 977.75 thousand faculty (or 977,750 faculty)

(g) Estimate the actual value in 2007 from the scatter plot. Include units.

1000 thousand

(h) What is the residual for 2007? Include units.

residual = actual − predicted

$= 4 - \hat{y}$

$=$ 1000 - 977.75 = 22.25 thousand faculty

(i) What is the *y*-intercept, including units? Explain what this means in a complete sentence in the context of this problem. Does this make sense in this context?

y-intercept $= 36.25(0) - 71776 =$ −71776

In the year 0 (two millenia ago), there were −71776 thousand faculty. This doesn't make sense.

(j) What is the slope, including units? Explain what this means in a complete sentence in the context of this problem.

Slope $= 36.25 \frac{\text{thousand faculty}}{\text{year}}$

If one more year passes, the number of faculty goes up by 36.25 thousand.

(k) According to the regression line, during what year will there be 1,100,000 faculty members? Show your work using the equation of the regression line. Is this extrapolation? Yes or no?

Solve $y = 1{,}100$ thousand (Give your answer to the nearest year.)

$36.25x - 71776 = 1{,}100$

$36.25x = 72876$

$x = \frac{72876}{36.25} \approx 2010$