Probability Theory

Marginal Distributions and Joint Moments

Given a joint density function for two random variables, X and Y, we can recover the individual density functions by summing (in the discrete case) or integrating (in the continuous case) over the other variable. The individual distributions are referred to as the __________________________________ .

In the discrete case:

$$f_X(x) = \sum_y f(x, y),$$

and

$$f_Y(y) = \sum_x f(x, y).$$

In the continuous case:

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy,$$

and

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx.$$

Note: The P Exam does not include continuous marginal distributions.

Ex: Below is the joint probability distribution for two discrete random variables, X and Y. Determine the marginal distributions for each variable and find $f(2,2) - f_X(2)f_Y(2)$.

$X \backslash Y$	1	2	3
1	0.15	0.10	0.16
2	0.25	0.12	0.13
3	0.08	0.01	0.00

Ex: Let X and Y be discrete random variables with joint probability function

$$p(x) = \begin{cases} \frac{2x+y}{12} & \text{for } (x,y) = (0,1),(0,2),(1,2),(1,3) \\ 0 & \text{otherwise} \end{cases}.$$

Determine the marginal probability function of X.

We can use joint densities to find expected values (and thus ____________________) for functions of two variables:

$$E[g(X,Y)] = \sum_x \sum_y g(x,y)f(x,y).$$

In particular, a common expected value we will need is

$$E[XY] = \sum_x \sum_y xyf(x,y).$$

<u>Ex:</u> The table below gives the joint probability function for a sailor's number of boating accidents and number of hospitalizations resulting from the accidents in a given year. Calculate the sailor's expected number of hospitalizations from boating accidents during the year.

		Number of Hospitalizations from Accidents			
Number of Accidents		0	1	2	3
	0	0.700			
	1	0.150	0.050		
	2	0.060	0.020	0.010	
	3	0.005	0.002	0.002	0.001

Ex: A couple takes out an insurance policy that reimburses them for days of work missed due to illness. Let X and Y represent the days of work missed in a month for the wife and husband, respectively. The policy pays a benefit of 50 times the maximum of X and Y, subject to a benefit limit of 100. X and Y are independent, each with uniform distribution on the set $\{0, 1, 2, 3, 4\}$. Calculate the expected monthly benefit paid to the couple.

Ex: Let X and Y be discrete random variables with joint probability function given below. Calculate $Var(Y)$.

$P(X = 1, Y = 1) = 0.10$
$P(X = 1, Y = 2) = 0.15$
$P(X = 2, Y = 1) = 0.18$
$P(X = 3, Y = 1) = 0.22$
$P(X = 3, Y = 2) = 0.05$
$P(X = 3, Y = 3) = 0.30$

Ex: Let X and Y be discrete random variables with joint probability function given

$y \backslash x$	0	1	2
0	0	$\frac{2}{5}$	$\frac{1}{5}$
1	$\frac{1}{5}$	$\frac{1}{5}$	0

below. Calculate $Var(Y - X)$.

Ex: Random variables X and Y have joint probability function $p(x, y)$ for $x = 0, 1$ and $y = 0, 1, 2$. Suppose $3p(1, 1) = p(1, 2)$, and $p(1, 1)$ maximizes the variance of XY. Calculate the probability that X or Y is 0.