Reading Guide for "Chromatic Number of Plane Tilings" paper

Directions: Use these questions as a guide to help you read *On the Chromatic Number of Plane Tilings* by D. Coulson. The full article is posted on Moodle. Try to answer the questions for the page you are on before moving on to the next page.

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1. If you have not worked with the chromatic number before, do an internet search for "Chromatic Number of Graphs" to see a simpler case than trying to find the chromatic number of the entire plane. Give a definition of a coloring problem in your own words.
2. What is the vertex set for the graph colored in this paper? How about the edge set? Explain why there is no picture of that graph.
3. Look at Figure 1, the Moser Spindle. (a) Pick a triangle and explain why 3 different colors would be necessary to label the vertices of that triangle. (b) After labeling the vertices of that triangle with three different colors, work through the rest of the vertices and explain why 4 colors are necessary to label all of the vertices in the Moser spindle.
4. Explain why the Moser spindle implies you need at least four colors to color the plane.
5. How does a hexagonal tiling imply seven colors is enough to color the plane with an excluded distance?
6. Look up the definition of a Voronoi region and write the definition in your own words.

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7. Summarize the main goal of this paper in your own words.
8. Draw at least two different examples of tilings that would NOT fit the conditions of a tiling considered in this paper.

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9. What method of proof is used to prove Lemma 3.1?
10. As you go through each piece of the proof, draw a picture to help you understand the statement (some of these may be the same as the pictures given in Figure 3.)
11. "By finiteness considerations..." What assumption about the type of tiling is being used here? You may need to go back to section 2.

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12. Lemma 3.2: "almost all" means all but a countable number (you can take finite number here). Since the tiles are polygons, explain why only a finite number of points on C can be on the boundaries of tiles rather than interior to the tiles?
13. The proof mentions that U cannot be colored a, b, or c. To see this, find points in U that are distance D away from points in the regions colored a, b, and c.
14. In the last paragraph of the proof of Lemma 3.2, it again uses "almost all points..." to make a claim. Explain why that condition can be used to find the necessary points colored d and e.
15. Continue in the same way with the statements and proof of each remaining theorem. Draw diagrams/pictures at each step of the proof to help your understanding.