"In several ground school lessons, the reader will learn the mathematical and physical reasons soaring pilots fly at precise speeds as well as a bit about the exhilarating and challenging sport of soaring."

Optimal Soaring:
What Is the Best Speed to Fly?

Michael A. Lundin
Central Washington University

Sailplane pilots commonly fly hundreds of miles in aircraft fueled by the sun's energy alone. During cross country flights, they often climb over 10,000 feet and remain aloft all day. To fly long distances pilots circle in rising air or "lift," gaining potential energy as they pirouette. Then they convert altitude into speed (kinetic energy) as they glide toward their destination. See Figure 1. The strategy may seem simple: climb in lift and glide in sinking air. However, on cross country flights, where lift is never guaranteed, precise flying is essential. To maximize energy gain and minimize its loss seasoned pilots fly at precise speeds, depending on the presence of winds, lift, or sink. A bit of calculus, or even pre-calculus, can tell us about the optimal speeds to fly under these conditions. In several ground school lessons, the reader will learn the mathematical and physical reasons soaring pilots fly at precise speeds as well as a bit about the exhilarating and challenging sport of soaring.

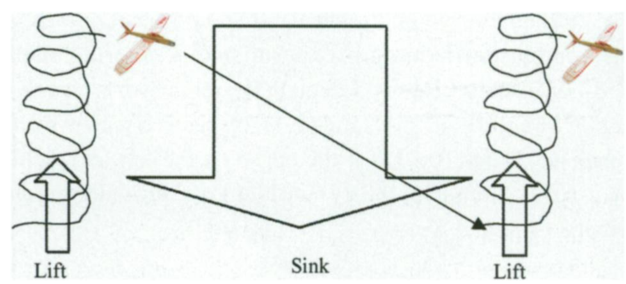
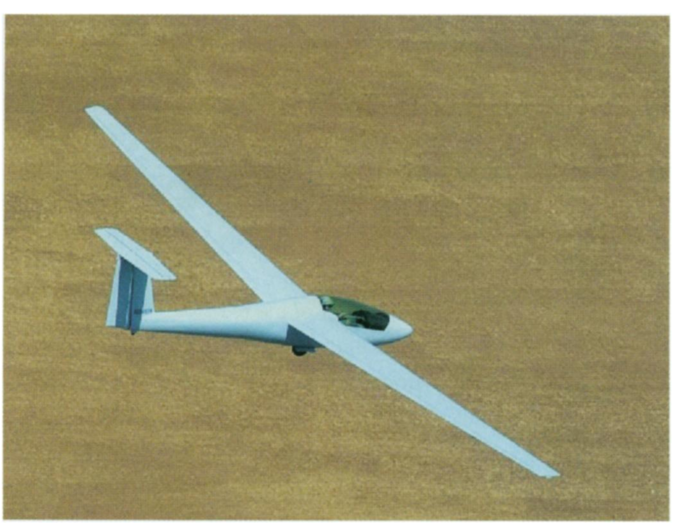


Figure 1. Slow down and circle in lift; speed up and glide in sink.



Ground School Lesson 1: How Gliders Fly

Soaring is elegant flying, like a 3D version of surfing. Just as a surfer rides a water wave, a sailplane rides a mound of air, but the mound itself may be falling, rising, moving laterally, or dead still. Again, as in surfing, mastering the game depends on reading the waves. Of course, air waves are invisible and that is part of the challenge!

In any case, flying safely is paramount. Expert pilots, while circling in lift, fly just fast enough to avoid stalling. Stalls happen when a glider's airspeed becomes too slow for the wings to lift the plane, so the plane drops. If not corrected, a stall at best wastes time en route, while at worst degrades into an unintentional, extremely bad landing! Pilots learn to prevent stalls by correctly reacting to a hushing wind noise, a displaced horizon, and to ominous instrument readings.

More mathematically, four quantities are important to glider pilots. The two most important quantities for pilots to know at any time are horizontal airspeed V_a and vertical air speed V_y. Pilots constantly monitor both quantities from cockpit instruments, a practice that promotes precise and safe flying. A third important quantity, unavailable to pilots except by calculation, is V_x or groundspeed, often conceived as the speed of the *shadow* of an airplane as it moves over the ground. Thus, the pilot flies from moment to moment by monitoring airspeed V_a, since the plane reacts to air movement. However, time en route and actual progress from landmark to landmark depend on ground speed. A fourth important quantity is glide ratio,

$$\frac{\text{ground speed}}{\text{vertical speed}} = \frac{V_x}{V_y}.$$

All gliders have inherent glide ratios, optimized values at particular airspeeds. Glide ratios may vary from less than 18/1 in low performance gliders to more than 50/1 in racing planes. Because glide ratios *change with airspeed*, it is useful to approximate them by

$$\frac{V_a}{V_y},$$

because numerator and denominator are instantly observable quantities, and because in still air, V_a closely approximates V_x. See Figure 2.

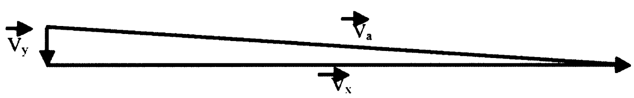


Figure 2. In still air, groundspeed and airspeed vectors have nearly the same magnitudes. The drawing is not to scale, given that glide ratios are larger than implied by the illustration. Approximately what glide ratio is indicated by the vector diagram?

Basically, good soaring technique means (1) flying slower in lift, and (2) speeding up to penetrate through sink. Surprisingly, perhaps, *different optimization techniques* provide the correct airspeeds in each situation. Demonstrating where these airspeeds come from is our goal here.

Ground School Lesson 2: Polar Curves and Flying Characteristics

Aeronautical engineers use complex mathematical models to design all aircraft, including gliders. The most important functions arising from those models are graphed as "polar" curves; that is, plots of V_y as a function of V_a. After a prototype aircraft is built, the polar curves are confirmed or "tweaked" by test pilots, who collect data while testing the plane's true performance. The reader may note that the misnomer, "polar," evolved from functions of the angle between incoming wind and the chord line through a wing cross-section, is only implicitly relevant here.

A polar curve is glider specific. Polar data plots for three popular types of sailplanes can be found in Figure 3, the Blanik L-13, the Schleicher ASW 22M, and the Rolladen-Schneider LS-3. Actual data are in Table 1.

Warm-up Question. Before reading further, the reader may find it instructive to rank the three sailplanes by performance, based on their polar profiles. How do natural sink rates (V_y), glide ratios

$$\frac{V_a}{V_y},$$

and relative penetration rates (sink rates at higher airspeeds) compare for the three sailplanes? If performance means price, and it does, could you rank the gliders by price? (Note that glide ratios

$$\frac{V_a}{V_y}$$

change as a function of V_a.) Finally, at what points on the graphs do stalls occur?

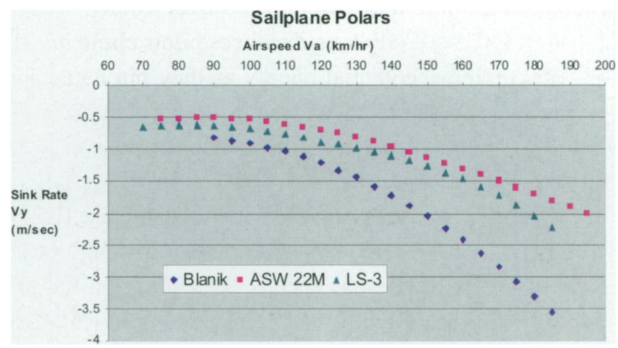


Figure 3. Polar curves for L-13, ASW 22M, LS-3.

Answer to Warm-up Question. The reader may have figured out that the best performer is the ASW 22M. At any given velocity, its sink rate is the least (in absolute value). The LS-3 comes in second, and the L-13, third. Penetrating power at higher airspeeds is consistent with the ranking; otherwise the curves implied by the data might converge or cross at higher airspeeds. Seeing that the ranking holds when comparing glide ratios is a bit trickier, and this is at the heart of the next section. For the reader's information, the Blanik L-13 (lowest performer) is a training glider ($15,000 used); the LS-3 (second best) is a midrange transition sailplane ($25,000 used); and the ASW 22M is a high performance speedster (at least $65,000 used). Stalls occur at airspeeds slightly less than those of the leftmost data points. (That is why the data appears to abruptly end on the left side of all three graphs; at those airspeeds the airplanes quit flying!) Our polar data came from the Australian Soaring Home Page and the prices are rough estimates, based on advertisements.

	V_y		
V_a	L-13	ASW 22M	LS-3
(km/hr)	(m/sec)	(m/sec)	(m/sec)
70			-0.66
75		-0.54	-0.64
80		-0.51	-0.63
85		-0.49	-0.63
90	-0.82	-0.5	-0.64
95	-0.86	-0.52	-0.65
100	-0.91	-0.54	-0.68
105	-0.97	-0.57	-0.71
110	-1.03	-0.6	-0.76
115	-1.12	-0.65	-0.82
120	-1.21	-0.69	-0.88
125	-1.33	-0.75	-0.92
130	-1.45	-0.8	-0.98
135	-1.58	-0.87	-1.04
140	-1.72	-0.96	-1.11
145	-1.88	-1.04	-1.18
150	-2.04	-1.13	-1.27
155	-2.23	-1.22	-1.36
160	-2.42	-1.31	-1.46
165	-2.63	-1.4	-1.59
170	-2.84	-1.5	-1.72
175	-3.07	-1.6	-1.87
180	-3.30	-1.69	-2.04
185	-3.56	-1.8	-2.22
190		-1.9	
195		-2.01	

Table 1: Sink Rates vs Airspeeds For Three Sailplanes.

Ground School Lesson 3: In lift, fly the sailplane at min sink, the airspeed that guarantees the minimal natural absolute sink rate.

In pilot's terms, when we have lift, we wish to lessen the glider's natural tendency to sink by flying at an optimal speed. How fast should we go? This is a typical problem for pre-calculus and calculus students. We will use data from the LS-3 for our calculations, and we invite the reader to use the data from Table 1 to perform similar calculations using data from the L-13 and ASW 22M. We use a graphing utility to model the function $V_y = f(V_a)$, for which a quadratic model works nicely! We find that

$$V_y = -1.47 \times 10^{-4}\, V_a^{\,2} + 2.44 \times 10^{-2}\, V_a - 1.65.$$

Using either pre-calculus or calculus methodology, we can approximate the vertex (local maximum) of the concave down parabola at the critical point (83.1,–0.64). See Figure 4. Hence, when there is lift, we wish to fly close to 83.1 km/hr, dubbed "*min sink*" by pilots, to minimize the natural downward travel of the glider, which will be –.64 m/sec. (The reader will find that a quadratic function also works well for the L-13 data, but a cubic function gives more realistic values for the ASW 22M.)

Ground School Lesson 4: In dead or sinking air, fly the sailplane at its best glide speed, which depends on the sink rate of the air.

Most beginning pilots erroneously think that flying at min sink, as discussed in Lesson 3, provides the longest glide distance, but *this is not so!* Flying at min sink merely keeps the plane aloft for the longest time. In contrast, flying the faster "max glide" airspeed maximizes the distance traveled, but max glide depends on the sink rate of the air itself. For simplicity's sake, we assume no horizontal air movement, a complexity we address later. To compute max glide we must optimize the glide function, noting that, in this case,

$$V_y = f(V_a) + S,$$

where $S \le 0$ is the downward speed of the air in which the glider is riding. Hence, the new glide ratio is

$$G(V_a) = \frac{V_a}{f(V_a) + S}.$$

To optimize G, one may take it's derivative with respect to V_a, setting the result to 0, assuming S is constant. More directly, if $f(V_a)$ and S are known, a graphing calculator can plot G and determine the optimal value of V_a. The reader is invited to optimize the ratio

$$G(V_a) = \frac{V_a}{V_y} = \frac{V_a}{f(V_a)} = \frac{V_a}{-1.47 \times 10^{-4} V_a^2 + 2.44 \times 10^{-2} V_a - 1.65},$$

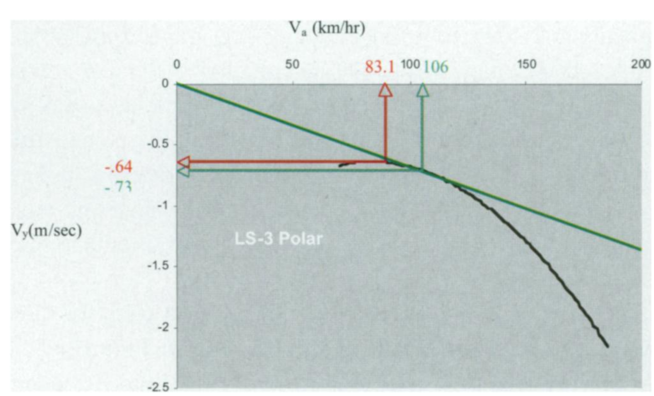


Figure 4. Every glider pilot learns to read polar curves such this one, which shows the LS-3 min sink airspeed and glider sink rate (red tangent) and the max glide airspeed and sink rate (green tangent).

the glide function for the LS-3, in still air ($S = 0$). Using calculus or pre-calculus methods, we can optimize this function to find the speed to fly $V_a^* = 106$ km/hr and natural glider sink rate at that speed to be $V_y^* = -.73$ m/sec. After making the units consistent we find a very good approximation for the best glide ratio for the LS-3 in dead air to be

$$\frac{V_y^*}{V_x^*} = -\frac{40}{1}.$$

The reader may note that in the technical literature glide ratios are always positive, the absolute values of their calculated quantities.

Finally, in our optimization we could have employed the quotient rule for derivatives, learned in elementary calculus. Setting that quotient to 0, we would have observed that

$$G'(V_a) = \left(\frac{V_a}{f(V_a)+S}\right)' = 0,$$

so

$$f'(V_a^*) = \frac{f(V_a^*)+S}{V_a^*}.$$

In particular, when $S = 0$, the above equation characterizes Figure 4, *which every beginning glider pilot learns to interpret*. A line tangent to the polar curve and through the origin has its tangent point at (V_a^*, V_y^*). In general pilots determine best glide airspeeds by drawing tangents to their glider's polar curve, from which they read the max glide airspeeds V_a^* and the glider's natural sink rates V_y^* for various values of S. When $S < 0$, pilots first move the origin of the graph up by S m/sec, then draw the tangent to the polar curve. Moving the origin up is the same as translating the polar curve down, which the reader may find more familiar and mathematically satisfying. Knowing this, when flying through sink, should a pilot fly the LS-3 faster or slower than 106 km/hr? Why? (Answer: The pilot should fly faster through sink. Physically, it is efficient to "zoom" through the pocket of sinking air, minimizing the loss of potential energy (height). To calculate the speed, the polar curve will translate down (origin goes up), so the point of tangency will slide to the right toward the faster airspeed.)

Ground School Lesson 5: In a tail wind, fly slower; in a head wind, fly faster, depending on the speed of the wind component.

When there is a horizontal wind speed component, the glide function takes on a new term H, either positive or negative, depending on whether the wind component is with or against the direction of flight. When wind is present, we have

$$V_y = f(V_a + H) + S$$

In this case the glide function becomes

$$G(V_a + H) = \frac{V_a + H}{f(V_a + H) + S}.$$

We leave it to the reader to compute the best glide speed in the LS-3 with a tail wind component $H = 10$ km/hr and an air sink rate $S = -2$m/sec. (Hint: Let $X = V_a + H$ and use the numerical features on a graphing calculator to find $X^* = 157.6$ km/hr, which implies that $V_a^* = 147.6$ km/hr.) Thus, in this case we fly faster to compensate for sink, but slower to compensate for the tail wind. It might now be clear to the reader that pilots calculate these speeds graphically via horizontal and vertical translations of the origin. They do so while planning their flights.

Of course much has not been stated! Adding weight to a glider shifts its polar curve toward higher airspeeds. Why do you think this happens? In fact, competition pilots add water ballast to their planes to increase speed for a given sink rate. Additionally, many competition pilots fly by the MacCready method, developed in the 1950s by an aeronautical engineer and world class glider pilot, Dr. Paul MacReady. Readers may recognize MacCready's name in conjunction with prize winning peddle-powered and solar-powered aircraft he designed. Dr. MacCready theorized that powerful lift conditions permit pilots to fly faster than they would otherwise, sacrificing more altitude for speed. The favorable lift conditions permit recouping spent energy. Other scientists and soaring enthusiasts have generalized MacCready's method. In particular, Helmut Reichmann wrote his doctoral dissertation, "On the Problem of Airspeed Optimization in Cross Country Soaring Flight" and later published the best guide on the subject, *Cross-Country Soaring*. One economist and soaring competitor, John Cochrane, has employed game theory and stochastic differential equations to outline winning strategies for timed soaring competitions.

Sailplane pilots now get real time information from GPS-fed onboard computers, in which algorithms output MacCready speeds. However, most world class competitors have their own techniques and continue to argue about competition winning strategies.

For more information on the sport of soaring, see the Soaring Society of America web site at `www.ssa.org`. Also, read a well-written and picture-laden blog of a world class soaring pilot's journeys through the Cascade Mountains of Washington at `http://mysite.verizon.net/reslezyt/aerial/vitek-re.htm`. ■