Homework for Chapter 11 Part 2 (11.9-11.11)

48 (a & b), 50, 51, 52, 53, 54, 60

Number 48

A.

$$E = -\frac{Z^2}{n^2}\left(\frac{e^4\mu}{8\epsilon_0^2 h^2}\right)$$

$$E = -\frac{9^2}{1^2}(2.178 \times 10^{-18}) = -1.765 \times 10^{-16}\, J$$

$$L = \sqrt{l(l+1)\hbar^2}$$

$$L = \sqrt{0(0+1)(1.054 \times 10^{-34})^2} = 0\, Js$$

$$L_z = m_l\hbar = (0)(1.054 \times 10^{-34}) = 0\, Js$$

B.

$$E = -\frac{Z^2}{n^2}\left(\frac{e^4\mu}{8\epsilon_0^2 h^2}\right)$$

$$E = -\frac{9^2}{3^2}(2.178 \times 10^{-18}) = -1.961 \times 10^{-17}\, J$$

$$L = \sqrt{l(l+1)\hbar^2}$$

$$L = \sqrt{2(2+1)(1.054 \times 10^{-34})^2} = 2.582 \times 10^{-34}\, Js$$

$$L_z = m_l\hbar = (2)(1.054 \times 10^{-34}) = 2.108 \times 10^{-34}\, Js$$

Number 50

$$E_H = -\frac{Z^2}{n^2}\left(\frac{e^4\mu}{8\epsilon_0^2 h^2}\right)$$

$$E_H = -\frac{1^2}{1^2}(2.178 \times 10^{-18}) = -2.178 \times 10^{-18}\, J$$

$$6.022 \times 10^{23} \times -2.178 \times 10^{-18}\, J = -1.312 \times 10^{6}\, J$$

$$E_{He} = -\frac{Z^2}{n^2}\left(\frac{e^4\mu}{8\epsilon_0^2 h^2}\right)$$

$$E_{He} = -\frac{2^2}{1^2}(2.178 \times 10^{-18}) = -8.712 \times 10^{-18}\, J$$

$$6.022 \times 10^{23} \times -8.712 \times 10^{-18}\, J = -5.246 \times 10^{6}\, J$$

Number 51

$$P = \int_a^b \Psi^*\Psi \, d\tau = \left(\sqrt{\frac{1^3}{\pi a^3}} e^{-\frac{r}{a}}\right)\left(\sqrt{\frac{1^3}{\pi a^3}} e^{-\frac{r}{a}}\right) \sin\theta \, r^2 d\phi d\theta dr$$

$$= \frac{1^3}{a^3\pi} \int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \int_{r=0}^{0.1} r^2 \, e^{-\frac{2r}{a}} \, dr$$

$$= \frac{1^3}{a^3\pi}\left[\phi\Big|_0^{2\pi}\right]\left[-\cos\theta\Big|_0^{\pi}\right]\left[e^{-\frac{2r}{a}}\left(\frac{-r^2a}{2} - \frac{ra^2}{2} - \frac{a^3}{4}\right)\Big|_0^{0.1}\right]$$

$$= \frac{1}{(0.529)^3\pi}\left[\phi\Big|_0^{2\pi}\right]\left[-\cos\theta\Big|_0^{\pi}\right]\left[e^{-\frac{2(0.1)}{(0.529)}}\left(\frac{-(.1)^2(0.529)}{2} - \frac{(0.1)(0.529)^2}{2} - \frac{(0.529)^3}{4}\right)\right.$$

$$\left. - e^0\left(\frac{-(0)^2(0.529)}{2} - \frac{(0)(0.529)^2}{2} - \frac{(0.529)^3}{4}\right)\right]$$

$$= (2.15)(2\pi - 0)(-(-1) - -(1))[(0.685)(-0.00265 - 0.0140 - 0.0370)$$

$$- (1)(0 - 0 - 0.0370)]$$

$$= (2.15)(2\pi)(2)[(0.685)(-0.0537) - (1)(-0.0370)]$$

$$= (27.0)[0.000216]$$

$$= 0.0058$$

Number 52

$$P = \int_a^b \Psi^*\Psi \, d\tau = \left(\sqrt{\frac{10^3}{\pi a^3}} e^{-\frac{10r}{a}}\right)\left(\sqrt{\frac{10^3}{\pi a^3}} e^{-\frac{10r}{a}}\right) \sin\theta \, r^2 d\phi d\theta dr$$

$$= \frac{10^3}{a^3\pi} \int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \int_{r=0}^{0.1} r^2 \, e^{-\frac{20r}{a}} \, dr$$

$$= \frac{1000}{a^3\pi}\left[\phi\Big|_0^{2\pi}\right]\left[-\cos\theta\Big|_0^{\pi}\right]\left[e^{-\frac{20r}{a}}\left(\frac{-r^2a}{20} - \frac{ra^2}{200} - \frac{a^3}{4000}\right)\Big|_0^{0.1}\right]$$

$$= \frac{1000}{(0.529)^3\pi}\left[\phi\Big|_0^{2\pi}\right]\left[-\cos\theta\Big|_0^{\pi}\right]\left[e^{-\frac{20(0.1)}{(0.529)}}\left(\frac{-(.1)^2(0.529)}{20} - \frac{(0.1)(0.529)^2}{200} - \frac{(0.529)^3}{4000}\right)\right.$$

$$\left. - e^0\left(\frac{-(0)^2(0.529)}{20} - \frac{(0)(0.529)^2}{200} - \frac{(0.529)^3}{4000}\right)\right]$$

$$= (2150)(2\pi - 0)(-(-1) - -(1))[(43.8)(-0.000265 - 0.000140 - 0.0000370)$$

$$- (1)(0 - 0 - 0.0000370)]$$

$$= (2150)(2\pi)(2)[(0.0228)(-0.000442) - (1)(-0.0000370)]$$

$$= (27000)[0.0000269]$$

$$= 0.726$$

Number 53

$$\#\text{ of radial nodes} = n - l - 1$$

$$\#\text{ of angular nodes} = l$$

A. Radial 1, Angular 0, Total 1
B. Radial 2, Angular 0, Total 2
C. Radial 1, Angular 1, Total 2
D. Radial 0, Angular 3, Total 3
E. Radial 1, Angular 4, Total 5
F. Radial 6, Angular 0, Total 6

Number 54

$$\int_{-\infty}^{\infty} \Psi^{*}\Psi \, d\tau = \left(\sqrt{\frac{2}{\pi a^3}}\left(2-\frac{r}{a}\right)e^{-\frac{r}{2a}}\right)\left(\sqrt{\frac{1}{\pi a^3}}e^{-\frac{r}{a}}\right)\sin\theta \, r^2 d\phi d\theta dr$$

$$= \frac{\sqrt{2}}{a^3\pi}\int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \int_{r=0}^{\infty} r^2 \left(2-\frac{r}{a}\right)e^{-\frac{3r}{2a}} dr$$

$$= \frac{\sqrt{2}}{a^3\pi}\int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \left(\int_{r=0}^{\infty} 2r^2 \, e^{-\frac{3r}{2a}} dr - \int_{r=0}^{\infty} \frac{1}{a} r^3 e^{-\frac{3r}{2a}} dr\right)$$

$$= \frac{\sqrt{2}}{a^3\pi}\int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \left(2\int_{r=0}^{\infty} r^2 \, e^{-\frac{3r}{2a}} dr - \frac{1}{a}\int_{r=0}^{\infty} r^3 e^{-\frac{3r}{2a}} dr\right)$$

$$= \frac{1}{a^3\pi}\left[\phi\Big|_0^{2\pi}\right]\left[-\cos\theta\Big|_0^{\pi}\right]\left[2\left(\frac{2}{\left(\frac{3}{2a}\right)^3}\right) - \frac{1}{a}\left(\frac{6}{\left(\frac{3}{2a}\right)^4}\right)\right]$$

$$= \frac{1}{a^3\pi}(2\pi - 0)(-(-1) - -1)\left[2\left(\frac{2}{\frac{27}{8a^3}}\right) - \frac{1}{a}\left(\frac{6}{\frac{81}{16a^4}}\right)\right]$$

$$= \frac{1}{a^3\pi}(4\pi)\left[2\left(\frac{16a^3}{27}\right) - \frac{1}{a}\left(\frac{96a^4}{81}\right)\right]$$

$$= \frac{4}{a^3}\left[\frac{32a^3}{27} - \frac{96a^3}{81}\right]$$

$$= \frac{4}{a^3}\left[\frac{96a^3}{81} - \frac{96a^3}{81}\right]$$

$$= \frac{4}{a^3}(0)$$

$$= 0$$

Number 60

$$P = \int_a^b \Psi^* \Psi \, d\tau = \left(\sqrt{\frac{1^3}{\pi a^3}} e^{-\frac{r}{a}}\right) r \left(\sqrt{\frac{1^3}{\pi a^3}} e^{-\frac{r}{a}}\right) 4\pi \sin\theta \, r^2 dr$$

$$= \frac{1^3}{a^3\pi} \int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi} \sin\theta \, d\theta \cdot \int_{r=0}^{\infty} r^3 \, e^{-\frac{2r}{a}} \, dr$$

$$= \frac{1^3}{a^3\pi} \left[\phi \Big|_0^{2\pi}\right] \left[-\cos\theta \Big|_0^{\pi}\right] \left[\frac{3!}{\left(-\frac{2}{a}\right)^4}\right]$$

$$= \frac{1^3}{a^3\pi} (2\pi - 0)(-(-1) - -1) \left[\frac{6}{\frac{16}{a^4}}\right]$$

$$= \frac{4\pi}{a^3\pi} \left[\frac{6a^4}{16}\right] = \frac{3a}{2} = \frac{3(0.529)}{2} = 0.794$$