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The Advantage of the Coin Toss for the New Overtime System in the National Football League

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On January 19, 2002, the Oakland Raiders faced off against the New England Patriots in the National Football League (NFL) playoffs. In a heavy snowstorm, the Raiders were up 13–3 at the beginning of the fourth quarter. Then, Tom Brady led the Patriots in one of his now famous comebacks. With less than a minute remaining in regulation, Adam Vinatieri successfully kicked a field goal that tied the game at 13. In the ensuing overtime period, the Patriots won the coin toss and took possession of the ball, which has been the typical strategy for decades. On their initial drive, the Patriots drove down the field and kicked a field goal for the win. The Raiders never took possession of the ball and thus did not have a chance to score. This game serves as an example of how many overtime contests in the NFL have ended, with a score on the initial drive by the coin toss winner. In fact, from 1994–2009, roughly 34% of coin toss winners won the game on their first possession. The possible advantage of winning the coin toss has been one of the more exciting and most criticized elements of the overtime system in the NFL.

The NFL instituted a sudden death format to overtime in 1974. (Before that, ties were allowed during the regular season and sudden death overtime was used during the playoffs.) Since the first team to score wins, the coin toss winner most often chose to receive the ball. In fact, only 11 times in NFL history has the coin toss winner decided to go on defense first in overtime. From 1974–1993, roughly 47% of coin toss winners went on to win the overtime period. In 1994, a rule change resulted in the kickoff being moved back five yards, from the 35 to the 30 yard line. From 1994–2009, almost 60% of coin toss winners went on to win the overtime period. In addition

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to empirical evidence, Jones examined the effect of the coin toss on NFL overtime using absorbing state Markov chains that yielded similar results [**3**]. Due to the well-documented advantage of winning the coin toss, the NFL has long discussed a possible change to the format of overtime.

In 2010, the NFL decided to implement a new overtime system with the following changes.

- If the team who possesses the ball first scores a touchdown, the game is over.
- If the team who possesses the ball first kicks a field goal, the other team has a chance to either win with a touchdown or tie with a field goal that would force the game into sudden death.
- If the team who possesses the ball first does not score, the game goes to sudden death.

This system was first implemented in 2011 during the playoffs only and fully implemented during the regular season in 2012. Since this new system has been in use for a very short amount of time, it is difficult to draw any definitive conclusions about the advantage of winning the coin toss. Therefore, extending the methods of [**3**], we propose a model to examine the effect of the coin toss on the new overtime system using the law of total probability and absorbing state Markov chains.

Absorbing state Markov chains

A *Markov chain* is a stochastic process X_n where

$$\begin{aligned} p_{ij} &= P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0\} \\ &= P\{X_{n+1} = j \mid X_n = i\}. \end{aligned}$$

The value p_{ij} represents the probability that the process will, when in state i, next make a transition into state j. In addition, p_{ij} does not depend on states prior to the ith state (this is known as the Markov property). An *absorbing state* of a Markov chain is a state that is impossible to leave once entered (i.e., $p_{ii} = 1$). An absorbing state Markov chain is one that contains at least one absorbing state where it is possible to reach an absorbing state from every other state that is not absorbing (called a *transient state*). For a Markov chain with r absorbing states and t transient states, the transition probability matrix can be written in the canonical block form

$$\left(\begin{array}{c|c} Q & R \\ \hline 0 & I \end{array}\right)$$

where Q is a $t \times t$ matrix giving the transition between transient states and R is $r \times t$ matrix giving transition probabilities from transient to absorbing states, with the $r \times t$ zero matrix and the $r \times r$ identity matrix giving transition probabilities from absorbing states.

An absorbing state Markov chain will reach an absorbing state with probability one. From any state that is not absorbing, the probabilities of absorption are given by

the matrix $\mathbf{B} = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{R}$ that has dimensions $t \times r$. The element b_{ij} of $\mathbf{B}$ is the probability that the chain starting in transient state i will end up in absorbing state j.

Preliminaries

As in [**3**], we assume that both teams are equally matched. This is a fairly reasonable assumption since the score at the end of regulation time was tied. For an offensive series, let α denote the probability of a touchdown (worth six points) , β the probability of a field goal (worth three points), and γ the probability of not scoring. (Prior to 2011, an overtime game has only ended twice with a safety. Therefore, we will consider the chance of a safety as negligible.) Therefore, $\alpha + \beta + \gamma = 1$. Due to factors such as field position, time remaining, and point deficit or surplus, the values of α, β, and γ will most likely vary during both the regulation and overtime periods. However, a simulation study showed that varying these values yielded results similar to those presented in this manuscript. Thus, we assume constant values for α, β, and γ for simplicity.

Again following [**3**], we label each state in overtime as $[a\ b\ T]$ where a is the number of points scored in overtime by team A, b is the number of points scored in overtime by team B, and T is the team with possession of the ball, or as [A wins] or [B wins]. Irrespective of the overtime system, eventually the overtime period will end in a win for team A, a win for team B, or a tie. For a playoff game, a tie cannot occur. The game will go into multiple overtime periods until a team wins, as happened recently during the 2013 playoffs when the Denver Broncos and the Baltimore Ravens went to double overtime.

A model for the old overtime system

For the traditional sudden death or old overtime system, [**3**] considered the four states 1:[0 0 A], 2:[0 0 B], 3:[A wins], and 4:[B wins]. Figure 1 displays a directed graph of the possible transitions from state-to-state in sudden death. Notice that if teams do not score, then the Markov chain alternates between states 1 and 2. Once a team scores, the Markov chain moves to the appropriate absorbing state.

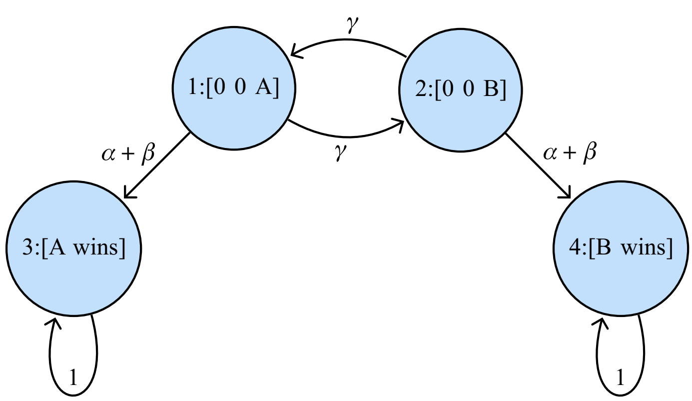

Figure 1. A directed graph describing the possible transitions from different states in the old overtime format.

Based on Figure 1, the one step transition matrix is

$$\mathbf{P}_{old} = \begin{bmatrix} 0 & \gamma & \alpha+\beta & 0 \\ \gamma & 0 & 0 & \alpha+\beta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Throughout this manuscript, we assume that team A wins the coin toss and hence receives the ball. Thus, in this setting, the probability that team A wins is the probability of starting in state [0 0 A] and eventually being absorbed into state [A wins]. If the overtime game occurred during the regular season, this probability would be the $(1, 3)$ element of $\mathbf{P}^n_{old}$ where n is the number of possessions in the overtime period. If the overtime game occurred during the playoffs, this probability would be the $(1, 1)$ element of the matrix of absorption probabilities,

$$\mathbf{B}_{old} = \begin{bmatrix} 1 & -\gamma \\ -\gamma & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha+\beta & 0 \\ 0 & \alpha+\beta \end{bmatrix} = \frac{1}{1-\gamma^2} \begin{bmatrix} \alpha+\beta & \gamma(\alpha+\beta) \\ \gamma(\alpha+\beta) & \alpha+\beta \end{bmatrix}.$$

As a result, team A wins with probability

$$\omega_{old} = \frac{\alpha+\beta}{1-\gamma^2} = \frac{1}{1+\gamma} \tag{1}$$

and team B wins with probability $1 - \omega_{old} = \gamma/(1+\gamma)$.

As an alternative to using Markov chains, ω_{old} can be derived by considering all the cases in which team A can win, such as scoring on their first possession, their second possession, and so on. Thus, team A wins with probability

$$\omega_{old} = \alpha + \beta + (\alpha+\beta)\gamma^2 + (\alpha+\beta)\gamma^4 + \cdots = (\alpha+\beta)\sum_{i=1}^{\infty} \gamma^{2i}$$

which is the sum of an infinite geometric series and simplifies to (1). For a regular season overtime period, a finite geometric series suffices.

A model for the new overtime system

Under the new overtime system, the chance of winning on future drives is dependent upon the outcome of the first drive, violating the Markov property. For example, the chance of moving from state [0 0 A] to state [A wins] is α on the initial possession of overtime. If team A does not score on their initial possession, the chance of moving from state [0 0 A] to state [A wins] is now $\alpha + \beta$. Therefore, the chance of moving to the state [A wins] depends on the outcome of the first drive. Furthermore, assuming that team A wins the coin toss and takes possession of the ball, we have

$$\begin{aligned} P(\text{A wins}) &= P\left[(\text{TD} \cap \text{A wins}) \cup (\text{FG} \cap \text{A wins}) \cup (\text{NS} \cap \text{A wins})\right] \\ &= P(\text{TD})P(\text{A wins} \mid \text{TD}) + P(\text{FG})P(\text{A wins} \mid \text{FG}) \\ &\quad + P(\text{NS})P(\text{A wins} \mid \text{NS}) \end{aligned} \tag{2}$$

by the law of total probability where TD is the event of a touchdown on the first drive, FG a field goal on the first drive, and NS not scoring on the first drive. Under the new overtime rules, the team with the initial possession of the ball wins if they score a touchdown on the first drive. Thus, $P(\text{A win} \mid \text{TD}) = 1$ and (2) become

$$P(\text{A wins}) = \alpha + \beta\, P(\text{A wins} \mid \text{FG}) + \gamma\, P(\text{A wins} \mid \text{NS}). \tag{3}$$

To find $P(\text{A wins} \mid \text{NS})$, we simply consider the possible states that occur after the initial possession team does not score. Essentially, if the team A gets the ball first and does not score, then the overtime period will go into sudden death with team B getting the ball "first." For an overtime period during the regular season with n possessions, $P(\text{A wins} \mid \text{NS})$ is the (2, 3) element of $\mathbf{P}_{old}^{n-1}$. For a playoff game, $P(\text{A wins} \mid \text{NS})$ is equivalent to the probability of starting in state 2 and ending in state 3 of sudden death, which is the (2, 1) element of the $\mathbf{B}_{old}$ matrix. Hence, for a playoff game,

$$P(\text{A wins} \mid \text{NS}) = \frac{\gamma}{1+\gamma}. \tag{4}$$

To find $P(\text{A wins} \mid \text{FG})$, we consider the five possible states that can occur after team A kicks a field goal on the first drive, namely 1:[3 0 B], 2:[3 3 A], 3:[3 3 B], 4:[A wins], and 5:[B wins]. Clearly, the chain will begin in state 1 with team B having possession of the ball and team A up three points. Figure 2 displays a directed graph of the possible transitions between these five states for the new overtime system if team A kicks a field goal on the first drive.

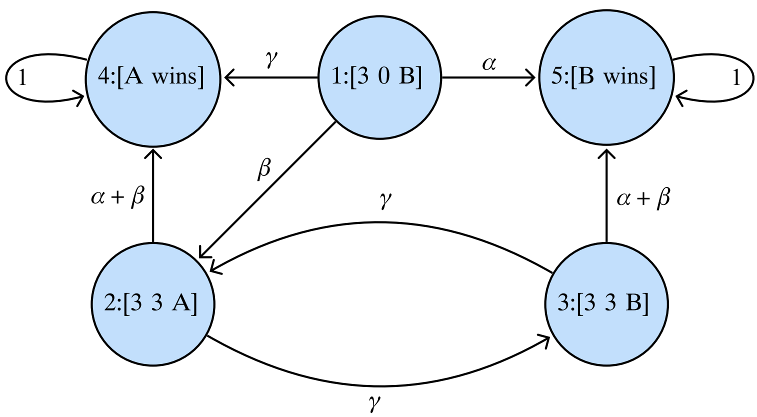

Figure 2. A directed graph describing the possible transitions from different states in the new overtime format after a field goal was made on the initial possession.

From Figure 2, the transition matrix is

$$\mathbf{P}_{new} = \begin{bmatrix} 0 & \beta & 0 & \gamma & \alpha \\ 0 & 0 & \gamma & \alpha+\beta & 0 \\ 0 & \gamma & 0 & 0 & \alpha+\beta \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

If there are n possessions in the overtime period, then $P(\text{A wins} \mid \text{FG})$ is the $(1, 4)$ element of the $\mathbf{P}_{new}^{n-1}$ matrix. For a playoff game, $P(\text{A wins} \mid \text{FG})$ will be the $(1, 1)$ element of the absorption probability matrix

$$\begin{aligned}\mathbf{B}_{new} &= \begin{bmatrix} 1 & -\beta & 0 \\ 0 & 1 & -\gamma \\ 0 & -\gamma & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma & \alpha \\ \alpha+\beta & 0 \\ 0 & \alpha+\beta \end{bmatrix} \\ &= \frac{1}{1-\gamma^2} \begin{bmatrix} \gamma(1-\gamma^2)+\beta(\alpha+\beta) & \alpha(1-\gamma^2)+\beta\gamma(\alpha+\beta) \\ \alpha+\beta & \gamma(\alpha+\beta) \\ \gamma(\alpha+\beta) & \alpha+\beta \end{bmatrix}.\end{aligned}$$

Therefore, if team A kicks a field goal on their first drive during an overtime playoff period, then the probability that they will eventually win is

$$P(\text{A wins} \mid \text{FG}) = \frac{\gamma(1-\gamma^2)+\beta(\alpha+\beta)}{1-\gamma^2} = \gamma + \frac{\beta}{1+\gamma}. \tag{5}$$

Substituting (4) and (5) into (3), the probability that team A wins a playoff game is

$$\omega_{new} = \alpha + \beta\left(\gamma + \frac{\beta}{1+\gamma}\right) + \gamma\left(\frac{\gamma}{1+\gamma}\right). \tag{6}$$

Comparing the two systems

We compare the two systems by first showing that $\omega_{new} < \omega_{old}$. Using $\alpha + \beta + \gamma = 1$ to eliminate α from (6) gives

$$\omega_{new} = \frac{1 - \beta\left[1 - (\beta + \gamma^2)\right]}{1+\gamma}.$$

Since $0 < \beta + \gamma < 1$, we have $0 < \beta + \gamma^2 < 1$. Thus, $0 < \beta\left[1 - (\beta + \gamma^2)\right] < 1$ and

$$\omega_{new} = \frac{1 - \beta\left[1 - (\beta + \gamma^2)\right]}{1+\gamma} < \frac{1}{1+\gamma} = \omega_{old}.$$

Since $\omega_{new} < \omega_{old}$, the new overtime system reduces the advantage of winning the coin toss.

To examine the possible magnitude of reduction in each the regular season and the playoffs, we will consider an example. To further compare the two overtime formats, we will need to obtain estimates of α, β, and γ. During the 2012 regular season, there was a total of 5,801 offensive possessions in which 1,163 of them resulted in a touchdown and 852 resulted in a field goal. Assuming the teams are evenly matched, we have

$$\alpha = \frac{1163}{5801} \approx 0.20 \ \text{ and } \ \beta = \frac{852}{5801} \approx 0.15$$

which leaves $\gamma \approx 0.65$.

We now consider an overtime period during the regular season, which can end in a tie. During the 2012 regular season, there were a total of 256 games. This yielded an average of 22.66 offensive possessions per game and 5.67 per quarter. For our comparison, we will assume a six possession overtime period. For the old overtime format, we have

$$\mathbf{P}_{old}^{6} = \begin{bmatrix} 0.075 & 0 & 0.560 & 0.364 \\ 0 & 0.075 & 0.364 & 0.560 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

By looking at the first row, team A wins with probability 0.560 and loses with probability 0.364. The game ends in a tie with probability 0.075. For the new overtime format, we have

$$\mathbf{P}_{old}^{5} = \begin{bmatrix} 0 & 0.116 & 0.560 & 0.324 \\ 0.116 & 0 & 0.324 & 0.560 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{P}_{new}^{5} = \begin{bmatrix} 0 & 0.027 & 0 & 0.725 & 0.249 \\ 0 & 0 & 0.116 & 0.560 & 0.324 \\ 0 & 0.116 & 0 & 0.324 & 0.560 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

So, if there are five possessions after the initial drive, then $P(\text{A wins} \mid \text{FG}) = 0.725$ and $P(\text{A wins} \mid \text{NS}) = 0.324$ since this chain begins with team B having the ball. Substituting these values into (6), team A wins the overtime period with probability 0.519. Similarly, the probability that team B wins is

$$\begin{aligned} P(\text{B wins}) &= \beta\, P(\text{B wins} \mid \text{FG}) + \gamma\, P(\text{B wins} \mid \text{NS}) \\ &= (0.15)(0.249) + (0.65)(0.560) = 0.401, \end{aligned}$$

and the probability of a tie is

$$\begin{aligned} P(\text{tie}) &= \beta\, P(\text{tie} \mid \text{FG}) + \gamma\, P(\text{tie} \mid \text{NS}) \\ &= (0.15)(0.027) + (0.65)(0.116) = 0.079. \end{aligned}$$

Using (1) and (6), we see that the probability that team A wins is 0.606 under the old format and 0.567 under the new format if an overtime game was played during the playoffs in which a tie cannot occur.

During both the regular season and the playoffs, it is clear that the new overtime format reduces the effect of the coin toss on the outcome of the overtime period. In addition, the chance of a tie during the regular season is similar between the two formats. Furthermore, for $0.05 \leq \alpha \leq 0.4$ and $0.05 \leq \beta \leq 0.4$, the probability of a tie between the two formats does not differ by more than roughly 1.6%. The largest differences occur when α is very small and β is very large.

Concluding remarks

In summary, the new overtime system reduces the effect of the coin toss when compared to the old format. However, the coin toss winner still has an advantage.

Other overtime formats have been suggested by the NFL and others that could also reduce the effect of the coin toss.

- At one time, the NFL discussed a “first to six” format where the first team to score six points in overtime wins [**2**]. Jones examined this format and found that it also reduced the effect of the coin toss but increased the chance of the overtime period ending in a tie by several percentage points [**3**].
- Based on empirical evidence, sudden death overtime was fair for the most part prior to 1994, when the kickoff was moved from the 35 yard line to the 30. Therefore, moving it back or even closer to the 50 yard line could possibly reduce the effect of the coin toss. However, this could come at a price since a higher percentage of overtime games during this period ended in a tie as opposed to after 1994.
- Che and Hendershott discussed eliminating the coin toss completely [**1**]. In lieu of a coin toss, each team would bid for their initial field position. The team who bid the furthest distance from scoring would win and take possession at that yard line.
- An anonymous reviewer suggested giving each team possession of the ball at least once irrespective of the outcome of the first drive.

It would be quite interesting to investigate the effect of the coin toss in each of these suggested overtime formats. No matter what suggestions or modifications are made, any overtime system will receive an equitable amount of praise and criticism by the media and fans alike.

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Summary. In the National Football League, the coin toss winner has had a clear advantage during an overtime period since 1994. The NFL instituted a new overtime system during the 2011 playoffs and the 2012 season in hopes of reducing the advantage of winning the coin toss. To analyze this change, with only a few seasons of data, we use absorbing state Markov chains to create a model and examine the effect of the coin toss in this new system.

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