Calibrating the Cobb-Douglas production function

Suppose the economy is characterized by the production function $Y = zK^{\alpha}N^{1-\alpha}$

Assume all markets are perfectly competitive and always in equilibrium.

The marginal product of capital is

$$\frac{\partial Y}{\partial K} = \alpha z K^{\alpha-1}N^{1-\alpha} = \alpha z K^{\alpha}N^{1-\alpha}/K = \alpha\frac{Y}{K}$$

A negative exponent implies division: $K^{-1} = {}^{1}/_{K}$. Then $K^{\alpha-1} = K^{\alpha}K^{-1} = {}^{K^{\alpha}}/_{K}$. This also suggests a non-standard way to think about the "power rule" of differentiation:

$$A = B^{c} \Rightarrow \frac{dA}{dB} = c\frac{B^{c}}{B} = c\frac{A}{B}$$

Using the same logic, the marginal product of labor is

$$\frac{\partial Y}{\partial N} = (1-\alpha)\frac{Y}{N}$$

In a closed economy without no taxes and no other factors of production, all earnings go to labor or capital, so

$$Y = wN + rK$$

where w is the wage earned by labor and r is the wage ("rental rate") or capital.

If factors are paid their marginal products, earnings by labor and capital are

$$\text{Earnings by labor} = wN = (1-\alpha)\frac{Y}{N}N = (1-\alpha)Y$$

$$\text{Earnings by capital} = rK = \alpha\frac{Y}{K}K = \alpha Y$$

In other words, the Cobb-Douglas production function implies that factors of production each earn a constant fraction of GDP. We can calibrate α by looking at data on the fraction of GDP being earned as wages and salaries. α is that fraction. If wages and salaries make up 65% of GDP, we should use $\alpha = 0.65$. See the LABSHPUSA156NRUG series from FRED for an example of estimates of this value:

https://fred.stlouisfed.org/series/LABSHPUSA156NRUG