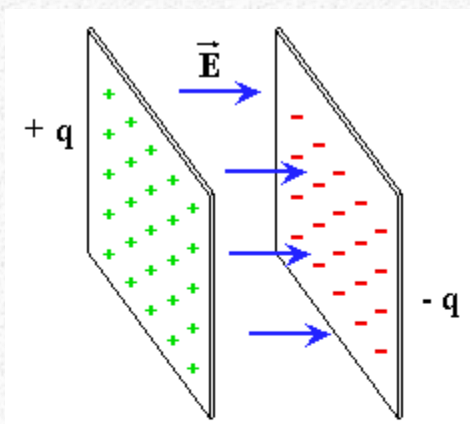

At this juncture, we pass over from rather abstract considerations to downright practical ones involving electrical parts such as wires, batteries, circuits, light bulbs, toasters, and so forth. If you *really* understand electric potential in the abstract sense we've been discussing (e.g., moving charges around in electric fields and seeing how the potential changes), it will be much easier for you to appreciate what's going on with real devices (e.g., why sticking a fork into an electrical outlet is not a good idea.) Apart from appreciating what electric potential is and how it influences charges, the key point you should keep in mind is this:

- The potential difference between any two points is independent of the path between them.

Note that this immediately implies that the potential difference around any closed circuit is zero. That is, if the starting and ending points are identical, there can be no change in potential.

# Capacitors and capacitance

Separated charges store energy, which we call electrostatic potential energy. As a purely pragmatic matter, how can we exploit this fact for fun and profit? The simplest device for storing energy by separating charges is a capacitor, so named for its capacity to store separated charges. The practical configuration and construction designs of capacitors vary greatly, but they all work on basically the same principle. Two conductors (generically referred to as "plates") are separated by an insulating layer that prevents the charge from flowing directly between them. Positive charge is placed on one conductor, while an equal and opposite charge is loaded on the other. The capacitor holds positive charge on one plate and an equal but opposite amount of charge on the other plate, so the net charge is zero. The charges produces an electric field that makes the two plates attract one another.




Recall that within a conducting material, there can be no electric field so the potential must have the same

value throughout. Therefore, any conductor is an equipotential, so each plate of a capacitor is an equipotential. From the geometry of the plates and the amount of charge stored there we could determine the potential difference between them simply by applying the same kind of arguments we covered last time. We'd have to work out exactly where the charge resides on the surface and then calculate how much energy it would take to move it from the other plate to where it sits now. This would be a mess for anything but a simple geometry. (We'll shortly discuss one simple example to make this more concrete.)

A more practical way to describe the capacitor follows from a powerful observation: The electric field at every point is proportional to the magnitude of the equal and opposite charges stored on the plates, so the *shapes* of the field lines do not depend upon the magnitude of the charges. Likewise, the electric potential between any two points is also proportional to the charge stored on the plates, so the *shapes* of the equipotentials also do not depend upon the magnitude of the charges. Therefore, if you double the charge on each plate, the field doubles everywhere and the potential difference between any two fixed positions doubles as well. Changing the net charge on all conductors in a region by some common factor merely relabels the potential value on every equipotential surface without shifting their positions or changing their shapes.

This observation implies that the total potential difference $\Delta V$ between the two capacitor plates varies directly with the amount of charge Q stored on them. In other words,

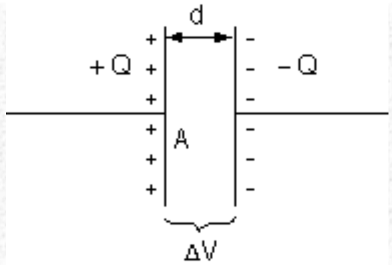
- For any capacitor of fixed geometry, $\Delta V$ and Q are directly proportional. Therefore, their ratio is a constant that characterizes all the construction details.

This parameter $C = Q/\Delta V$ is called the capacitance and is measured in Farads, where 1 Farad is defined to be 1 Coulomb/Volt . (Again, note how all these units are built up using the simplest combinations of previously defined units. New names for certain combinations of units are for convenience, they rarely indicate anything physically novel. Building from meters, kilograms, and seconds, the only fundamentally new quantity we've had to introduce this semester is the Coulomb. That was necessary because electric charge expresses a fundamentally new characteristic about the universe, something distinct from space, mass, and time.)

So capacitance measures an object's capacity for storing equal and opposite charges when a given voltage difference is applied. The bigger the capacitance, the more charge on each plate (though the net charge is always zero). The easiest way to determine capacitance is by reading what's printed on its side (how's that for straightforward?). If that doesn't work, one usually takes the empirical route by carrying out some simple experiment and making measurements. Only in rare moments, as in an introductory physics class, does one resort to theoretical considerations to derive the capacitance from scratch.

## A rare moment

Consider a capacitor constructed from two flat plates, situated parallel to each other. Their flat faces have surface area A, and they are separated from one another by some much smaller distance d. The thickness of the plates is irrelevant because basically all charge accumulates on the side that faces the other plate! If we deposit equal and opposite amounts of charge on the plates, the charge will be drawn to the surface facing the other plate. The perfectly matched charges are at rest across an impassible divide from each other, like a sixth-grade dance where equal numbers of boys and girls sit in long lines of folding chairs on either side of a gym. (I warned you that this subject stimulates a lot of bad writing.) Here's a two-dimensional schematic of such a capacitor:




The horizontal lines attached to the vertical plates represent conducting wires. Anyway, here's an important fact to remember:

- The electric field between a pair of flat, parallel, charged plates is approximately uniform

That's analogous to the fact that the gravitational field from a large flat plane of mass (which is what you usually imagine the Earth to be like) is approximately uniform so long as you stay close to the surface. The approximation is good anywhere whose distance to the nearest edge is large compared to the distance between the plates . In the picture above the field **E** points toward the right. Its magnitude, $E$, depends only upon the charge per unit area $\sigma = Q/A$ distributed across their faces. The full equation is $E = 4\pi k_e\sigma = \sigma/\varepsilon_o$ where using $k_e = 1/4\ \pi\varepsilon_o$ is another common way of writing Coulomb's constant. (This version simply absorbs certain ubiquitous factors of $4\pi$ to make equations simpler.) The separation between plates doesn't make a difference to $E$ so long as $d$ is much smaller than the width and breadth of the plates.

A constant electric field is useful, because it eliminates a pesky integration and makes potential drop $\Delta V$ just the field magnitude $E$ times the net distance $\Delta x$ moved along the field direction: $\Delta V = E\ \Delta x$. This holds everywhere between the two charged plates. Therefore, because all the positive charges are a distance $\Delta x = d$ from the negative plate, they have an electric potential difference of $\Delta V = E\ d$ relative to that other plate. Equivalently, moving a positive charge $q$ through a distance $d$ from the negative to positive plate requires exerting some mechanical work $W$ against the constant electric force $F_E = qE$, i.e., $W = qE\ d$. This work increases the electrical potential energy $\Delta U_E$, and the change in electrical potential itself is just $\Delta V = \Delta U_E/q = E\ d$. Either way, substituting for $E$ gives $\Delta V = Q\ d/A\varepsilon_o$. By comparing this with the definition of capacitance ($C = Q/\Delta V$), we find that $C = A\varepsilon_o/d$ for a parallel-plate capacitor.

(Note: I recommend that you work through the logic of that derivation yourself until it makes sense. While you won't be expected to reproduced it, there's something important in each individual step, so understanding each of them is a good sign of your comprehension.)

This example is useful because it indicates the basic factors of any capacitor. For one thing, it shows that $\Delta V$ is indeed proportional $Q$ for this geometry, just as our general argument predicted. The dependence on plate separation $d$ reflects the fact that pulling unlike charges away from each other requires energy and hence increases the system's electrostatic potential energy. That is, $E$ doesn't depend upon $d$ but $V$ does. Finally, for a given potential difference, increasing the area perpendicular to the electric field allows additional charge to be placed in a similar state. There really aren't any other factors to consider apart from the universal constant $\varepsilon_o$. Hence, we could have guessed this answer without going through the formal analysis.

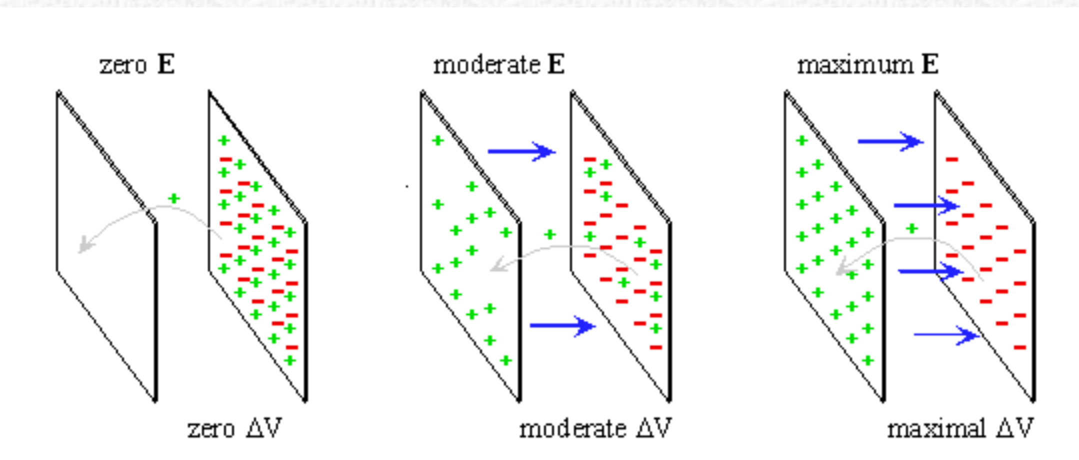
**Warning**: Do not confuse the electric force between two parallel plates with the electric force between two point charges. The electric field does not diminish with increasing distance from a charged flat plate, as it does from a point charge. Increasing the separation of two parallel plates does not change the electric field between them, though it does increase their potential difference, just as it does for oppositely charged point charges.

## Energy storage

A capacitor stores separated charges. Implicit in that is that a capacitor also stores energy. After all, you have to supply work at some point to bring them into their position on the opposite faces of the capacitors! This is similar to the way that raising objects stores gravitational potential energy, with one major difference: when you move ordinary massive objects around (books, barbells, six-packs, etc.), you don't significantly alter the gravitational field.

With charged objects, each quantity of charge moved from one place to another *does* alter the electric field. If we consider the capacitor above, when there is no net charge on the plates, the potential difference between them is zero. Therefore, the first minuscule bit of charge moved from one plate to the other requires essentially no energy. In contrast, just before the capacitor is fully charged to its final potential $\Delta V_{\text{final}}$, the electric field is almost at its maximum, and it takes a lot of energy to move that charge.

This figure illustrates that. Each "+" indicates a small quantity of positive charge moved across the divide in the process of charging up the capacitor. (In practice, nothing ever passes directly across the gap between the plates. However, charge can be taken along a different route via wires, but fortunately, the change in potential doesn't depend upon the path, only on its endpoints!) Let each infinitesimal bit have an amount of charge equal to $dq$. The total amount of charge transferred in $N$ steps is just $Q = N*dq$. However, the total energy required is not $Q$ times the final voltage because most of the charge is moved across before the potential drop got so large. The energy required increases as more charge is accumulated on the far side, so denoting the energy required to move bit number $i$ by $dU_i$, we have $dU_1 < dU_2 < dU_3 < ... < dU_{N-2} < dU_{N-1} < dU_N$.




If the left frame, moving the first bit of charge takes no energy whatsoever, because the potential hasn't built up yet, so $dU_1 = 0$. The next bit of charge would involve some small amount of energy because the first $dq$ would have established its own weak field. The potential continues to increase as charge is transferred, as each bit of charge already transferred only makes it tougher for the charge remaining. In the right frame, moving the final bit of charge requires the maximum amount of energy because the potential is essentially at its final, maximum value, so $dU_N = dq*\Delta V_{\text{final}}$.

To recap, the amount of energy stored in the capacitor $U$ equals the total amount required to separate the charges as we've described. For each $dq$ transferred, the amount of energy varies between zero (at the beginning) and $dU_N = dq*\Delta V_{\text{final}}$ (at the end). Because the potential increases steadily as charge is transferred, the average energy is simply halfway between those extremes, $dU_{ave} = (1/2)\ dq*\Delta V_{\text{final}}$. Counting up the energy for $N$ total steps gives the total energy as $U = N*(1/2)\ dq*\Delta V_{\text{final}} = (1/2)\ Q\ \Delta V_{\text{final}}$. Using the definition of capacitance, $C = Q/\Delta V$, this can be rearranged to give $(1/2)\ Q^2/C$.

Note the distinction between $U = 1/2\ Q\ \Delta V$ on a capacitor and $\Delta U = q\ \Delta V$ for the energy gained by a test particle of charge $q$ moving through a fixed potential of magnitude $\Delta V$, which is caused by other charges, not the test particle! In a capacitor, the charge $Q$ is what causes $\Delta V$, and the energy $U$ is simply the total energy stored. Hence, because "Δ" means "from one place to another" in the electrostatic context, it isn't appropriate for denoting the energy of a capacitor.

Summary: in a capacitor with charge $Q$ at potential $\Delta V$, the energy stored is simply half what it would take to separate the stored charge through a constant potential difference of that magnitude. The factor of 1/2 accounts for the fact that $\Delta V$ actually increases gradually from zero to its full value as charge is incrementally added.

## Circuits

Our next task will involve connecting capacitors together using long, narrow conductors, usually referred to as wires. They are often confined within insulating sheathes, and a collection of electrical devices connected by wires is called a circuit. You'll see a lot of wires in the coming weeks, both in the lab and as cryptic symbols on the page. It takes some facility to become good at translating between the two. The nice thing about wires is that for many purposes the precise physical configuration is unimportant. A wire can be twisted, knotted, drooped, flopped, or tangled with little or no influence on the rest of the circuit. You'll often find it more important to arrange things to facilitate your own comprehension rather than by any authoritative rules.

The key thing to remember about wires is this:

- Wires are treated as very good conductors, i.e., in practice they are equipotentials.

This is what makes potential so useful. Whenever you can trace a wire from one thing to another, you know that the pieces connected together have the same potential.

Another development is that we'll soon consider circuits in which charges are in continual motion. This modifies one important truth in electrostatic situations (i.e., those where charges don't move): All conductors are no longer required to be strictly equipotentials. By continually supplying charge at one end of a conductor and removing it from the other end, we can make an electric field within the material persist, which implies a drop in potential across the conductor. The relation between this potential drop and the rate at which charge moves across it depend upon the microscopic details of how individual charged particles pass through the material. At one extreme are insulators, which don't allow charge to flow at all under ordinary circumstances. At the other extreme are superconductors, which remain equipotentials even when currents flow through them. As usual, most of life lies between the extremes.