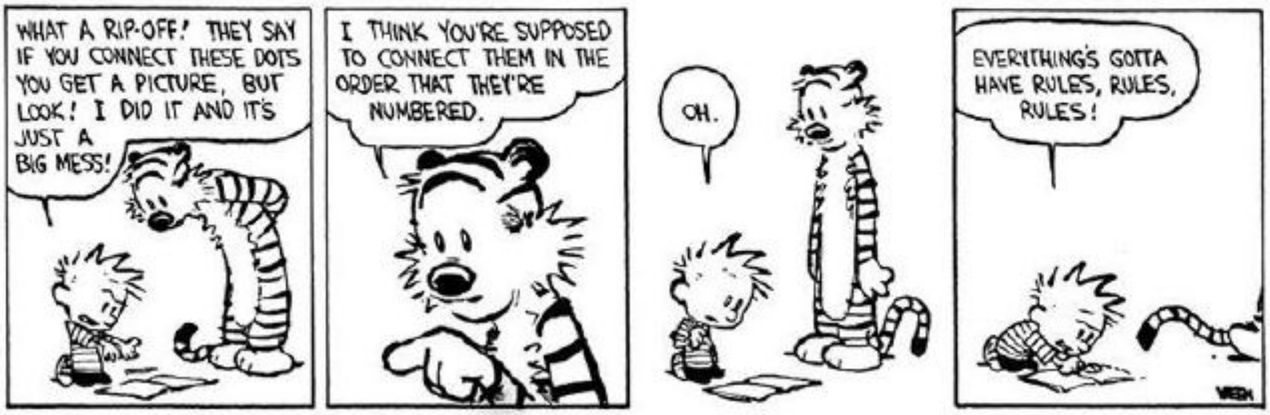
Class 12



Combining capacitors

When all charges have settled down and aren't moving around (i.e., there's no current), you are guaranteed that any conductor is an equipotential. This fact is very useful and applies to any combination of obvious conductors that are physically connected by other conductors, including simple wires. In contrast, the amount of charge on connected conductors is **not** generally the same on different components that are connected together. Moreover, charge is not equitably distributed in any simple sense. The distribution of charge depends in a complicated way on the physical configuration of the conductors.

For example, let there be a net non-zero amount of charge on a conducting sphere (left frame below). The surface charge density σ (charge per area - Greek lower-case letter sigma) will be perfectly uniform if there are no other sources of electric fields around, but that's a very special, highly symmetric situation. If an external field is applied, enhancements of charge accumulate on opposite sides of the sphere, so the surface charge density is very nonuniform. However, in both situations every point on the surface shares the same value of potential, i.e., the surfaces are equipotentials.

When you're examining combinations of capacitors, look for plates that are connected together. Any such pair or group of plates are at the same potential. However, by itself this observation does not imply that the potential drop across the two plates of those capacitors are identical! You've merely made a conclusion about one side of each capacitor so connected. If the other plates are also connected together, then they also share a common potential, which would indeed imply that the potential drops are indeed the same. The guiding principle for electrostatic problems is this:

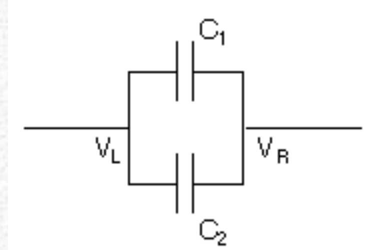
- Electric potential makes the rules, electric charges follow them

That is, always look first to see what conductors are connected and use that constraint to determine potential differences before asking how charges respond. There are some simple algebraic equations for combining capacitors, which are presented below. However, if you understand the source of those rules, you'll find them

much easier to apply and you'll be much less likely to screw up.

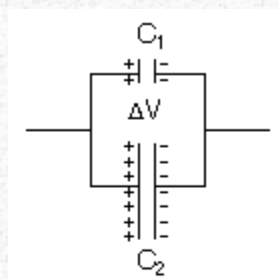
Capacitors in parallel

Here's a diagram of involving two capacitors connected so that they share the same potential drop. As in the schematic diagram in the last notes, the parallel lines next to the C's symbolize the two sides of a capacitor. You're welcome to visualize any capacitor in a circuit as a pair of parallel plates, although physical capacitors can have any manner of ridiculous shapes. Fortunately, those details are all absorbed into a single parameter for each capacitor, the capacitances C_1 and C_2. The conducting wires show us which plates share the same potential and indicate how these might be joined to other circuit elements. For this arrangement, both right plates have some potential V_R and both left plates have some potential V_L, though the two values are generally different. Therefore, the potential differences (or drop) $\Delta V = V_L - V_R$ between the left and right plates are the same for both capacitors (don't worry about the sign for now).



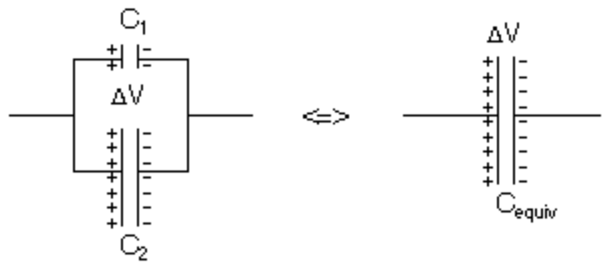
Circuit elements such as capacitors are attached in circuits at each end, usually via a pair of wires sticking out of them. A group of elements joined as described above is said to be connected in parallel. Everything in the group has one side connected together, so those ends are all at some shared potential. All the other ends are also connected together, so they are all also at one potential. The key thing about being in parallel is that each element has the same potential drop.

How do the charges compare for capacitors in parallel? We can answer this algebraically by looking at the definition of capacitance: $C = Q/\Delta V$ so $Q = C\,\Delta V$. An ensemble of capacitors in parallel stores more charge than any individual capacitor, but they all have the same potential difference. Therefore, the capacitance C of the whole must be larger because ΔV is the same but the total Q is increased. Okay, but let's take a more physical look at this conclusion. Visualize each capacitor as a pair of parallel plates, both pairs having the same separation. From what we know about parallel capacitors, a larger capacitance for the same separation requires plates with a larger area.



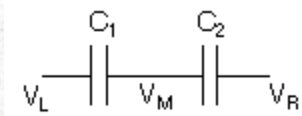
Schematic figures don't usually illustrate that difference, they just label the capacitor symbols with various subscripted C's and let you make the appropriate conclusions. For the sake of explanation, this diagram makes it explicit via the difference in the plate sizes that C_2 is much greater than C_1. Because the potential drop across each capacitor is the same and the plate separation is the same, the electric field strength between the plates is also the same. Therefore, there are more electric field lines between the two plates of larger area. Field lines originate and terminate on charges on the plate, so the plates with larger areas must have more charge on them. Same thing that algebra told us, but this give a better picture to carry around in our heads.

So what is the net effect of combining a bunch of capacitors in parallel, each with a difference capacitance C_i? The potential drop across each individual capacitor is the same, while the total charge Q is simply the sum of that on each capacitor Q_i. Add up all the charges Q_i and express each as $C_i\,\Delta V$, then factor out the shared potential drop: $Q = (C_1 + C_2 + C_3 + \ldots)\,\Delta V$. This equation is identical to the general equation that defines capacitance via the relation between charge and potential drop. All that's necessary is identifying the capacitance parameter. Doing so, we find that this arrangement could be replaced by a single capacitor of capacitance $C_{equiv} = (C_1 + C_2 + C_3 + ..)$ because it would store that same total amount of charge Q at the same potential drop V.



Graphically, combining capacitors in parallel is just like combining the areas of several parallel-plate capacitors with identical plate separations. Because capacitance is proportional to plate area, the capacitance simply adds together directly.

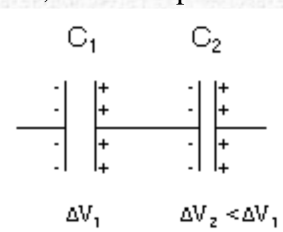
Capacitors in Series



The alternative way to connect capacitors together is in series. Here's the schematic for this arrangement with the potentials labeled on each connecting wire. Only the two plates in the middle are connected now (the right plate of C_1 and the left plate of C_2), so they share a potential of V_M. Nothing more is known about the other two potentials V_L and V_R. However, because the two connected plates aren't connected to anything else in the circuit, their net charge can be assumed to be zero. One could artificially place excess charge on the pair, but circuits are usually not designed that way. In other words, for the two connected plates, one can be charged only at the expense of the other because they are isolated from the rest of the circuit. A net charge of + Q on the right plate of C_1 thus implies a net charge of - Q on the left plate of C_2. The charges on them would flow back together across the wire between them and cancel each other unless there are potential differences that prevent that.

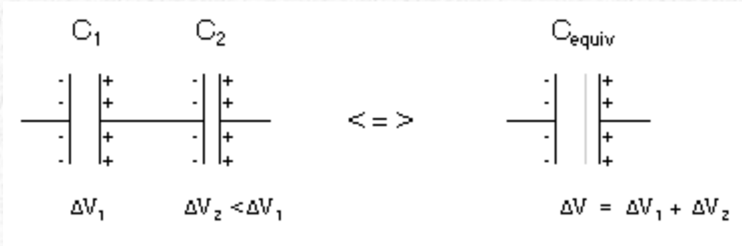
A continual separation requires a potential differences across each capacitor, but the differences $\Delta V_1 = V_L - V_M$ and $\Delta V_2 = V_M - V_R$ are not equal. What about the charges on the exterior plates, i.e., the left plate of C_1 and the right plate of C_2? A capacitor in a circuit generally has equal magnitudes of charge on its plates. Therefore, because the two isolated plates each have charge +/- Q, so will the outer plates. For several capacitors joined in series, each stores the same amount of separated charge. However, the charge stored on the interior plates is unconnected to the rest of the circuit and essentially unavailable for any other purposes.

How do the potential drops compare for capacitors in series? An ensemble of capacitors in series stores no more charge than any individual capacitor in the group, but the net potential difference across the entire group is larger than across any single element. Therefore, the capacitance $C = q/\Delta V$ of the whole must be smaller because Q is the same but the total ΔV is increased. Again, that's kind of abstract, so let's repeat our exercise with pictures. V isualize each capacitor as a pair of parallel plates, both pairs with the identical plate sizes. The charges are the same and so are the surface charge densities, which means that the electric fields between the plates are identical. Because the plate separation d is smaller for C_2, potential difference $\Delta V = -E\,d$ is smaller there.



We can use this understanding to determine how combined capacitors behave just as before. The potential drops are not the same, but the total potential drop ΔV is sum of that across each capacitor ΔV_i. Add up all the potential drops ΔV_i and express each as Q/C_i, then factor out the shared charge: $V = Q(1/C_1 + 1/C_2 + 1/C_3 + ...)$. Once again, this equation is identical to the general equation that defines capacitance via the relation between charge and potential drop, though what we identify as the capacitance parameter is different. Inspection shows that this arrangement could be replaced by a single capacitor with inverse capacitance $1/C_{equiv} = (1/C_1 + 1/C_2 + 1/C_3 + ..)$ because it would store an amount of

charge Q at the same total potential drop ΔV. In the figure below, the equivalent capacitor is represented by more widely separated plates but the same charge, which indicates a larger total potential drop.



Graphically, combining capacitors in series is just like combining the separations of several parallel-plate capacitors with identical plate sizes. Because capacitance is inversely proportional to plate separation, the capacitance adds together reciprocally.

Summary

Okay, here's the easily memorizable summary (though, of course, I discourage memorization!)
For capacitors in parallel:

- potential drop across each is the same
- total charge stored is the sum of that on all the individual capacitors
- equivalent capacitance: $C_{\text{equiv}} = (C_1 + C_2 + C_3 + ...)$

For capacitors in series:

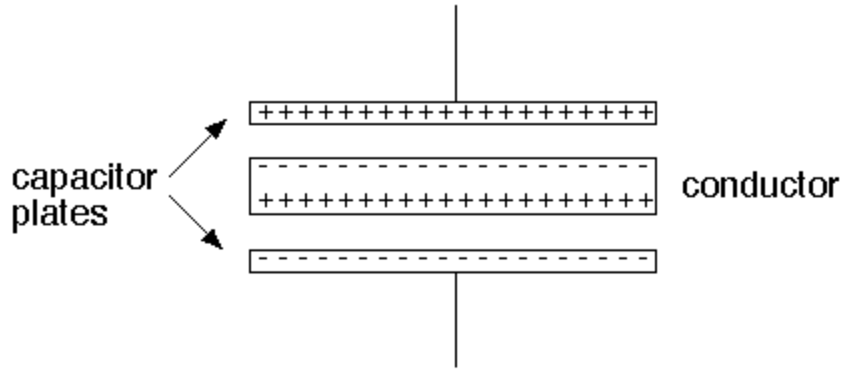
- charge stored on each is the same
- total potential drop is the sum of that across each of the individual capacitors.
- equivalent capacitance: $1/C_{\text{equiv}} = (1/C_1 + 1/C_2 + 1/C_3 + ...)$

Circuit elements can be combined in quite elaborate looking arrangements. The usual strategy is to identify small, simple combinations and replace them with equivalent elements. That simplifies the circuit, which allows further simple combinations to be identified, and so on.

Dielectrics

The last subject relevant to capacitor design are materials known as dielectrics. A dielectric is not really an insulator or a conductor. The internal charges can shift around but only a little. Electric fields are totally shielded out of a conductor, while an insulator does nothing to interfere with them. Dielectrics are intermediate in that they partially shield out an externally applied electric field. The practical result is that by separating a capacitor's plates with a dielectric rather than an insulator, the capacitance can be increased. That is, a smaller potential difference is needed to store a given amount of charge.

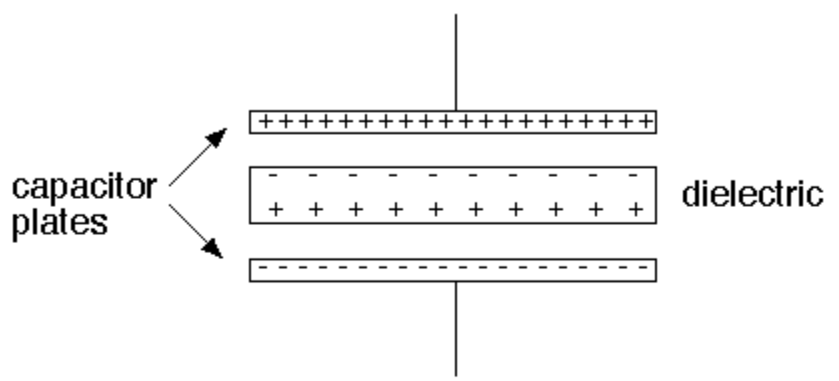
Consider first what would happen if we inserted another conductor between a capacitor's plates while still providing enough insulation to keep the separated charges from flowing across the gap. The insert would polarize, canceling out the field within itself. The surface charge on the polarized conductor would have the same density as on the capacitor plates, and the remaining electric field in the gap would be unchanged.



It might seem that this doesn't really change the situation. After all, the capacitor plates still have the same charge and produce the same electric fields. But it does affect the potential difference between the capacitor plates. Start at the bottom plate and calculate how much work you must expend to push a positive charge upward (note that the electric field points down). Until it gets to the conductor, the work required is exactly the same as if the insert weren't present. However, once across the lower boundary of the insert, the charge is shielded from the electric field produced by the plates. Therefore, it takes no energy to move it all the way through to the other side. Once that other face is crossed, it's an uphill climb again, but that intermediate plateau means that the total energy required to get to the other side has been substantially reduced.

Because the electric field between parallel plates is uniform, it follows immediately that the net potential drop between the plates has been reduced in proportion to the fraction of the separation now occupied by the conductor. Therefore, the same charge can be stored at a reduced potential drop. The only limit on how small that potential drop can be is how big a conductor you can insert before charges start jumping across whatever insulator fills the remaining space. However, a moment's consideration might lead you to a simple question: Why don't we just bring the plates together rather than precariously situating this additional conductor between them? The electric field is the same regardless of their separation, so potential is proportional to their separation.

The answer is that capacitors aren't usually modified in this way. However, a dielectric does something very similar. First of all, like an insulator, it doesn't allow free charges to pass through it, so it's a good material for separating plates. However, it can become partially polarized, which means that the electric field inside it is *partially* reduced by the accumulation of excess charge on its surface. The dielectric isn't an equipotential, but it tries its darndest to keep close to being an equipotential, not as well as a conductor would, but much better than an insulator.



So what happens when we push a positive test charge from the bottom to the top plate? The first part of the journey is the same as always, a persistent uphill struggle. Within the dielectric, the field is reduced, so you should think of this as a region of reduced slope. It's still uphill, but the going isn't quite as tough. In the final section, above the dielectric, it's back to the steep slope. The final result is that a smaller amount of energy is required overall, which indicates that the electric potential difference between the plates has been

reduced.

Although I've drawn the dielectric as separate from the plates to facilitate comparison with the inserted conductor, there's no reason it can't be totally flush against the plates and fill the entire gap. Indeed, this is typically the case. The effectiveness of the dielectric in reducing the potential and hence increasing the capacitance is expressed in an empirical parameter that depends upon the particular substance being used. The dielectric constant κ (Greek lower-case kappa) is defined by the equation $\Delta V = \Delta V_o / \kappa$ where ΔV is the potential drop with the dielectric inserted and ΔV_o is when it is absent but the same amount of charge is stored. This implies that $C = \kappa C_o$, where C and C_o are defined analogously.

With a dielectric present, you can treat the capacitor exactly as before in terms of the relation between charge Q and potential drop Δ just use C rather than C_o. Likewise, the equation for energy storage is unchanged.