Temperature and Thermal Expansion

Warmup Question

You’ve probably seen those small gaps between sections of roadway where Arkadelphia crosses the freeway. They allow the road to expand without buckling when the air temperature changes greatly. Try to estimate the coefficient of linear expansion for concrete based on that example. If that seems too difficult at first, consider what other quantities you need to estimate (or observe) first to make this determination. (Don’t get hung up on details or try to be too precise. Explain your chain of logic.)

ANS: The bridge is approximately $100\,\mathrm{m}$ long and has a larger than $0.5\,\mathrm{cm}$ gap at each end when the temperature outside is approximately 60°F. I'm going to assume, therefore, that joints on each end of the bridge will expand around $0.5\,\mathrm{cm}$ as the bridge heats up to 105°F on a very hot day.

Therefore, the bridge will expand approximately 1cm for a $45°\mathrm{F} = 25°\mathrm{C}$ change in temperature.

Therefore, the coefficient of linear expansion is the relative change in length is

$$\frac{\Delta L}{L} = \frac{0.01\,\mathrm{m}}{100\,\mathrm{m}} = 0.001 = \alpha \Delta T\ ,$$

where we assume a temperature change of 25°C. This gives us an expansion coefficient of

$$\alpha = \frac{\Delta L}{L\Delta t} = \frac{0.001}{25°\mathrm{C}} = 0.00004\,°\mathrm{C}^{-1}\ ,$$

or $4 \times 10^{-5}\ °\mathrm{C}^{-1}$.

This is around four times greater than the tabulated value. Not bad for rough guesstimates.

Warmup Question

In which of these temperature scales is a negative temperature physically impossible?

1. Fahrenheit
2. Celsius
3. Kelvin
4. Centigrade
5. All of the above
6. None of the above

ANS: **3**—You can't have negative temperature on the kelvin scale.

For Fahrenheit and Celsius (aka Centigrade), the zero of temperature is defined to be easily achieved in the laboratory (or even experienced on a cold day). In the kelvin scale, on the other hand, the zero of temperature is actually the lowest temperature you can achieve. (In reality, according to the Third Law of Thermodynamics, you can't even get to zero kelvin.) There is no temperature lower than 0 K.

Okay, what I said above is not *entirely* true. In statistical physics the temperature of a system is defined in a very precise mathematical way. It is possible to design a contrived system such that the absolute temperature actually is negative. However, the negative temperatures are "hotter" than positive temperatures! This is an interesting topic we cover in PH304–Thermal Physics. I think I've just given you enough motivation to take that class, no?

In physics class you will encounter a number of formulas involving Temperature, T, and change in temperature, ΔT. You will also encounter a number of temperature scales including Celsius, Fahrenheit, and Kelvin. In which of the formulas below is it allowable to express the temperature or change in temperature in Celsius or Farenheit degrees?

1. $Q = mc\Delta T$
2. $PV = nRT$
3. $\Delta L/L = \alpha \Delta T$
4. $P = \sigma A T^4$
5. Formulas 1 and 3
6. Formulas 2 and 4
7. All four formulas

ANS: **5**—Formulas 1 and 3 can use any temperature scale.

The main distinction between the role of temperature in these equations is that formulas 1 and 3 involve *change* in temperature, while equations 2 and 4 involve the *value* of the temperature. You can use any temperature scale for formulas involving ΔT, as long as you express the constants in the problem in appropriate units. However, when you use the value of temperature in a formula, that temperature must be on an absolute scale, like Kelvin or Rankine. Absolute temperature scales are the only ones where the value $T = 0$ has a real physical meaning.

Take equation 2, the ideal gas law, for example. The value $T = 0$ has a real physical meaning: this is the temperature at which a gas of fixed quantity and volume has zero pressure. If Celsius values were used in this formula, for example, it would imply that an ideal gas has no pressure when water freezes. Of course this is nonsense. The pressure of the gas should be zero only at absolute zero. Therefore, the value $T = 0$ can only occur at "absolute zero," which require the use of an absolute scale like kelvin (or Rankine).

When you are using a formula involving ΔT, however, the zero of temperature scale is ignored. For example, if you want to measure the thermal expansion of an object heated from 0°C to 10°C, $\Delta T = 10$°C. Equivalently, you would be heating the object from 273K to 283K, so $\Delta T = 10$K, which is the same quantity.

At how many distinct temperatures will the Fahrenheit and Celsius scales be equal?

1. None
2. One
3. More than one

ANS: **2**—There is only one temperature with the same values on the Fahrenheit and Celsius scales.

We know the following two points of information about the scales. The freezing point of water is $T_{\mathrm{C}} = 0°\mathrm{C}$ and $T_{\mathrm{F}} = 32°\mathrm{F}$. The boiling point of water is $T_{\mathrm{C}} = 100°\mathrm{C}$ and $T_{\mathrm{F}} = 212°\mathrm{F}$.

This means a change in temperature of 100°C corresponds to a change in temperature of 180°F, or $\Delta T_{\mathrm{F}}/\Delta T_{\mathrm{C}} = 180/100 = 9/5$. In fact, the ratio of differences between any two temperatures on these scales is $\Delta T_{\mathrm{F}}/\Delta T_{\mathrm{C}} = 9/5$. Therefore, using the freezing temp as a reference point, we have

$$\frac{T_{\mathrm{F}} - 32}{T_{\mathrm{C}} - 0} = \frac{9}{5} \quad \longrightarrow \quad T_{\mathrm{F}} = \frac{9}{5} T_{\mathrm{C}} + 32\,.$$

Let's solve this to determine for what value $T_{\mathrm{F}} = T_{\mathrm{C}}$:

$$T = \frac{9}{5} T + 32 \quad \longrightarrow \quad \frac{4}{5} T = -32 \quad \longrightarrow \quad T = -40\,.$$

Therefore, −40°F is equivalent to −40°C.

At how many distinct temperatures will the kelvin and Celsius scales be equal?

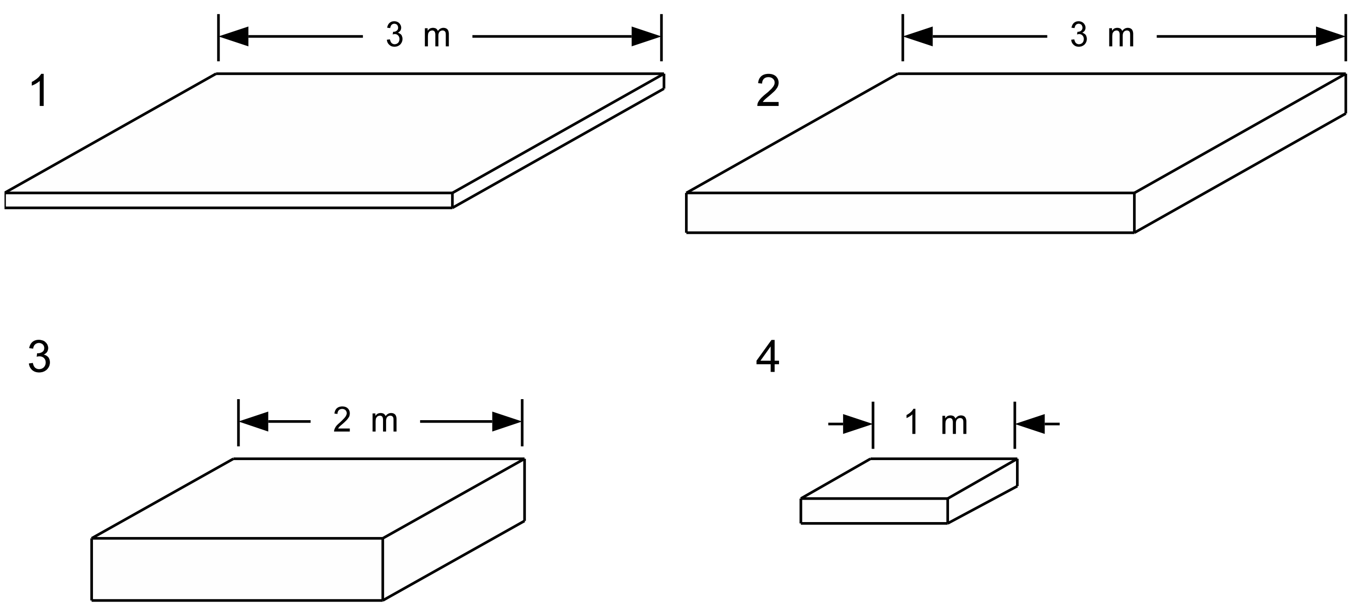
1. None
2. One
3. More than one

ANS: **1**—The temperatures will never be the same.

The value of the temperature on the kelvin scale is always 273.15 degrees higher than the corresponding temperature on the Celsius scale. They can never be equal.

The figure shows four metal plates with horizontal edges of 1, 2, or 3 meters, as indicated. All plates are made of the same material, and all their temperatures are to be increased by the same amount.

Which plate will experience the greatest increase in vertical height?



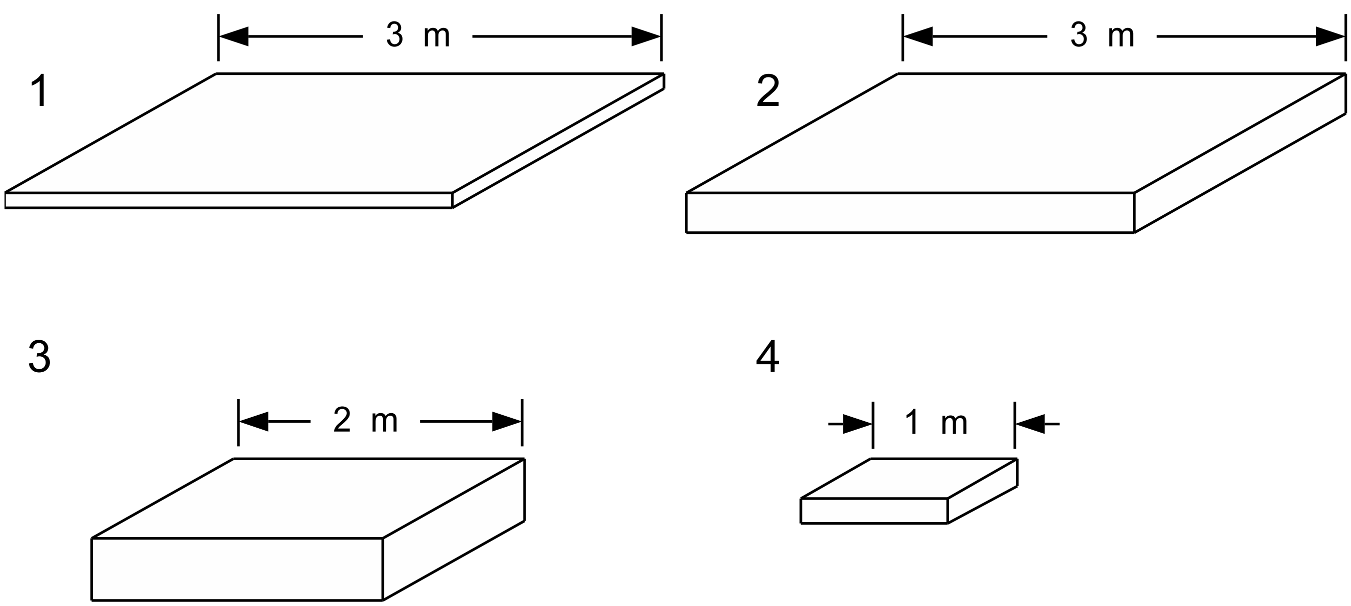
ANS: Plate **3** will experience the greatest increase in vertical height.

All plates are made of the same material, so all have the same relative increases, $\Delta L/L = \alpha \Delta T$. The actual increase in a linear dimension, ΔL, is equal to the relative increase multiplied by the initial size L. The bigger you start, the more you grow.

Plate 1 will have the smallest increase in height because it has the smallest height to begin with. Don’t fall into the trap of trying to think about the other two dimensions.

The figure shows four metal plates with horizontal edges of 1, 2, or 3 meters, as indicated. All plates are made of the same material, and all their temperatures are to be increased by the same amount.

Which plate will experience the smallest increase in volume?

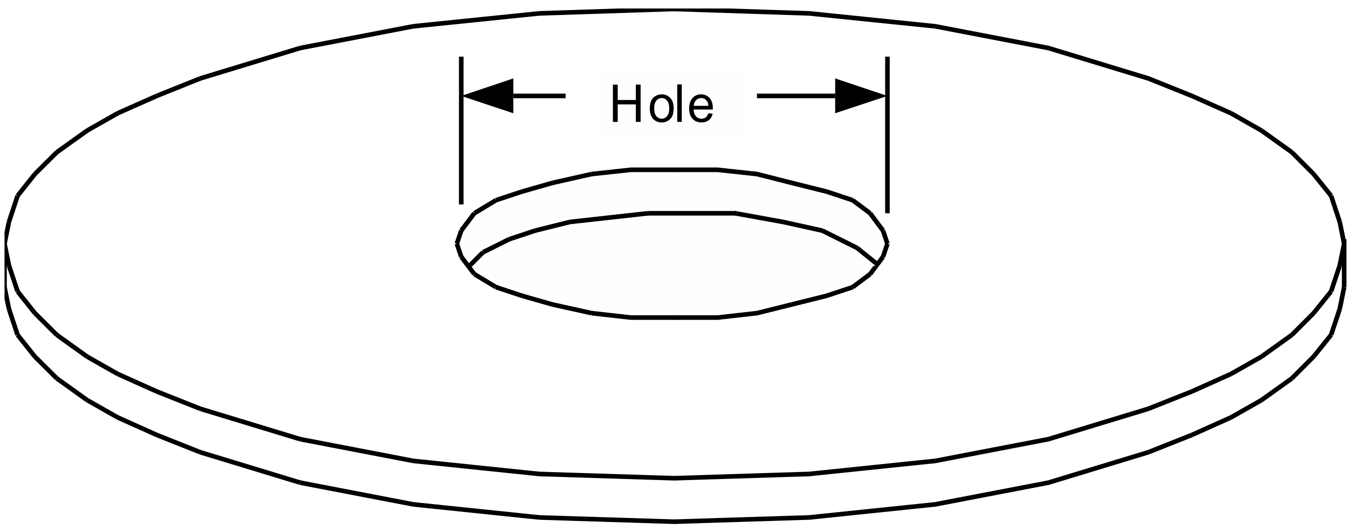


ANS: Plate **4** will experience the smallest increase in volume.

All plates are made of the same material, so all have the same relative increases in volume, $\Delta V/V = \beta\Delta T$. The actual increase in volume dimension, ΔV, is equal to the relative increase multiplied by the initial size V. The smaller you start, the less you grow. Plate 4 clearly has the smallest volume of all the plates.

Plate 2 will probably have the largest increase in volume. It's top surface area is equal to the area of plate 1, but plate 2 is thicker and therefore has a greater volume. Plate 2 has $9/4 = 2.25$ times the top surface area of plate 3. If plate 3 had 2.25 times the thickness of plate 2, they would have the same volume. However, plate 3 doesn't look quite that thick. It would appear that plate 2 has the greatest volume and will experience the greatest increase in volume.

A steel washer is heated until the metal expands substantially.

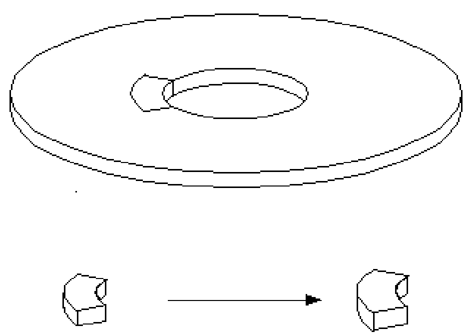


The diameter of the hole will

1. Increase.
2. Decrease.
3. Not Change.

ANS: **1**—The hole gets bigger.

The figure below outlines a small section of material along the inner rim and shows how it expands when heated. As the section of the washer warms up, it expands in every dimension. That means that the length of the inner arc of the section gets longer. Remember, however, that this section and a number of identical sections cut around the inner radius of the washer add up to form the entire inner circumference. If the inner arc of each of these sections increases, the total inner circumference will increase. This circumference remains circular, so the diameter of the hole must increase proportionally. Expansion is best viewed as a simple change of scale where everything gets bigger.



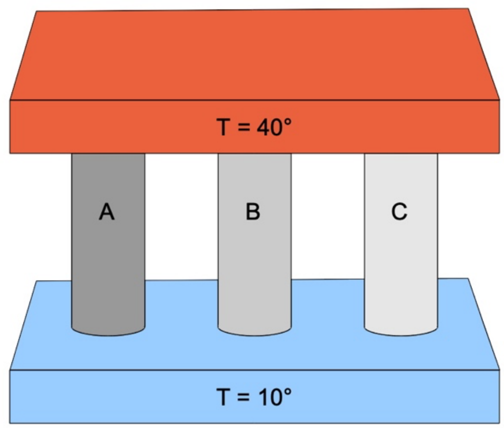
Here's a slightly different way to think about it. Suppose we had a solid steel disk, rather than a washer (the hole is filled in). As the disk is heated, it expands in all dimensions smoothly and without buckling or cracking.

Now suppose that we have the steel washer, but fill the hole with a steel disk. The small inner disk will certainly expand as it is heated. If the hole in the washer did not expand, eventually the washer and inner disk would press against each other. This would cause the washer/disk pair to crack or buckle. The only difference between this system and the large solid disk is a small circular line cut between the washer and inner disk. Could this line cause such radically different behavior? Nope. We are forced to conclude that the hole in the washer must increase in size.

Three metal cylinders with identical geometric dimensions are placed atop a heat reservoir that is maintained at 10°C. The heat conductivities are related as follows:

$$k_A = 2\,k_B$$
$$k_B = 4\,k_C$$

The tops of the cylinders are then placed into contact with a second heat reservoir at 40°C.



When the cylinders reach a state of thermal equilibrium with their respective reservoirs, what can you say about the heat conducted by the various cylinders?

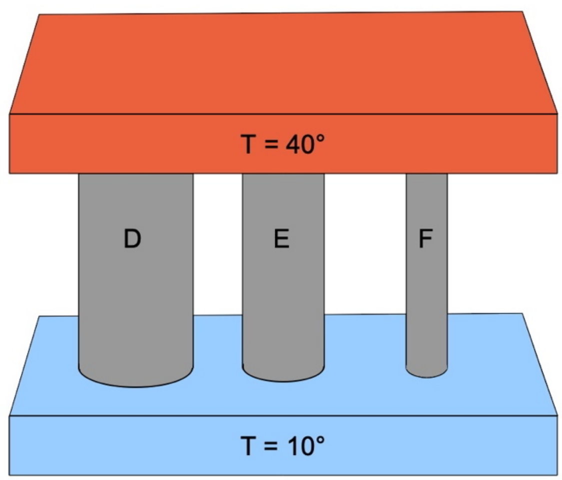
1. Cylinder A conducts the most heat
2. Cylinder B conducts the most heat
3. Cylinder C conducts the most heat
4. All cylinders conduct the same amount of heat
5. More information is needed

Answer: 1. Because the geometric dimensions are identical, the amount of heat conducted depends on the ratios of the heat conductivities. Cylinder A has the highest heat conductivity, therefore it conducts the most heat.

The original cylinders are now replaced by three new cylinders with different areas in contact with heat reservoir at 10°C, but the heat conductivities are the same. The areas of cylinder tops are related as follow:

$$A_D = 2A_E$$
$$A_E = 4A_F$$

The tops of the cylinders are again placed into contact with a second heat reservoir at 40°C.



When the cylinders reach a state of thermal equilibrium with their respective reservoirs, what can you say about the heat conducted by the various cylinders?

1. Cylinder D conducts the most heat
2. Cylinder E conducts the most heat
3. Cylinder F conducts the most heat
4. All cylinders conduct the same amount of heat
5. More information is needed

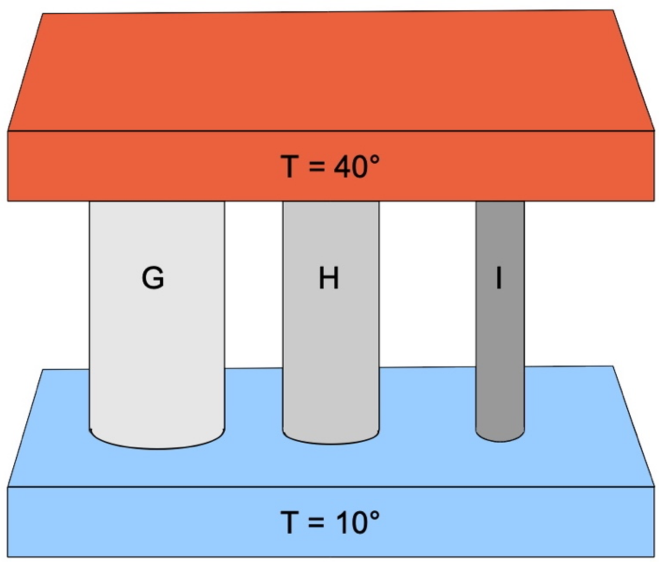
The answer is 1. The heat conductivities are identical, so the determining characteristics here are the areas in contact with the heat reservoirs. The cylinder with the largest area therefore conducts the most heat.

The cylinders are now replaced with three new cylinders whose dimensions are the same as in the last example. That is, the areas are related to each other by:

$$A_G = 2A_H$$
$$A_H = 4A_I$$

This time, the thermal conductivities are related as follows:

$$k_G = \frac{k_H}{2}$$
$$k_H = \frac{k_i}{4}$$



When the cylinders reach a state of thermal equilibrium with their respective reservoirs, what can you say about the heat conducted by the various cylinders?

1. Cylinder G conducts the most heat
2. Cylinder H conducts the most heat
3. Cylinder I conducts the most heat
4. All cylinders conduct the same amount of heat

5. More information is needed

The answer is 4. The ratio of the areas is exactly offset by the ratio of heat conductivities.

Finally, three cylinders are connected end to end as shown. The lengths are related to each other by the following:

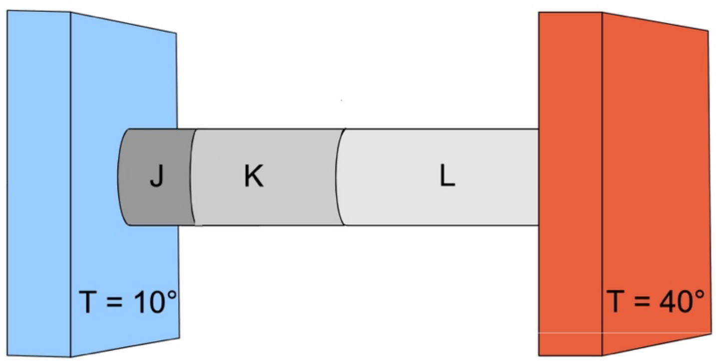
$$L_J = \frac{L_K}{2}$$

$$L_K = \frac{2}{3} L_L$$

The heat conductivities are related by

$$k_J = 2\, k_K$$

$$k_K = 2\, k_L$$



When the cylinders reach a state of thermal equilibrium with their respective reservoirs, what can you say about the heat conducted by the various cylinders?

1. Cylinder J conducts the most heat
2. Cylinder K conducts the most heat
3. Cylinder L conducts the most heat
4. All cylinders conduct the same amount of heat
5. More information is needed

The answer is 4. In a state of thermal equilibrium, the amount of heat exiting each cylinder must equal the amount entering the one next to it.