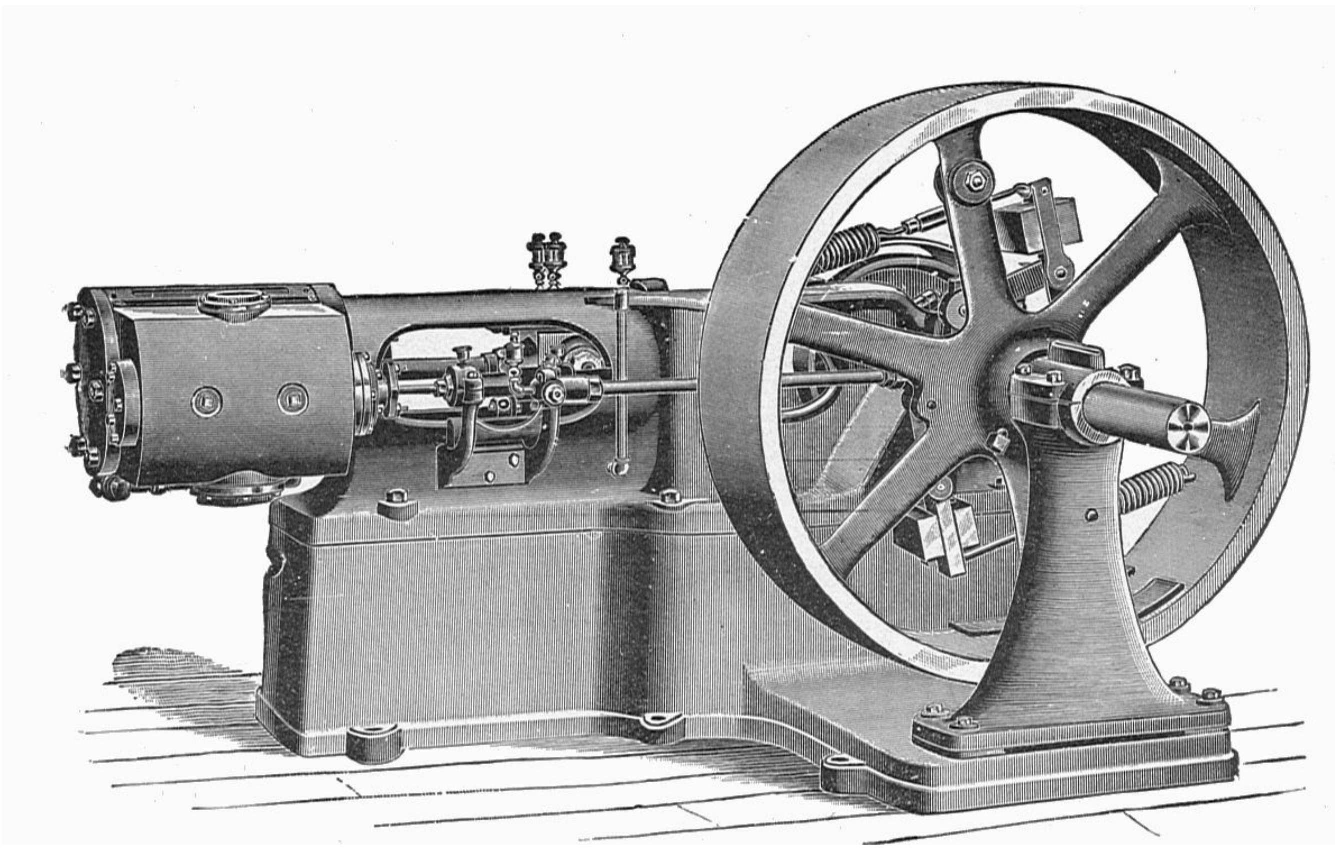
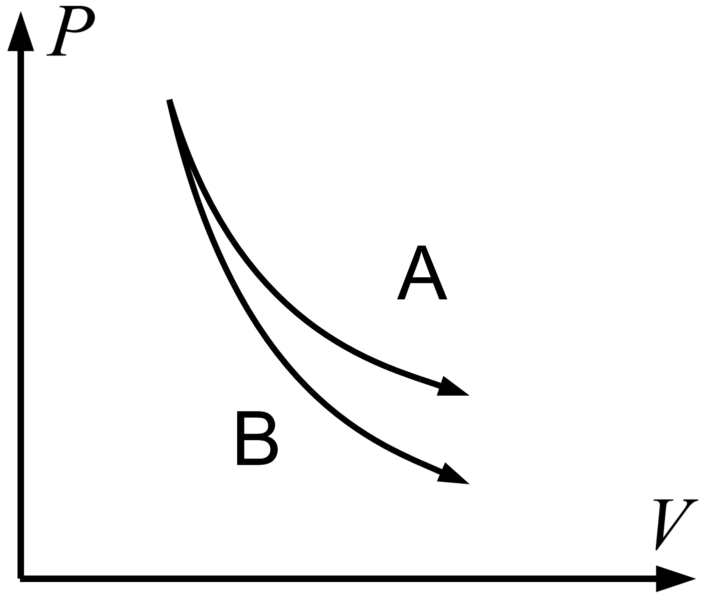
Heat Engines



Two identical, ideal gases start from the same thermodynamic state. One expands adiabatically, the other isothermally, as shown. Which is the adiabatic path?



1. Curve A
2. Curve B
3. More information is needed

ANS: **2**—Curve B is the adiabatic path.

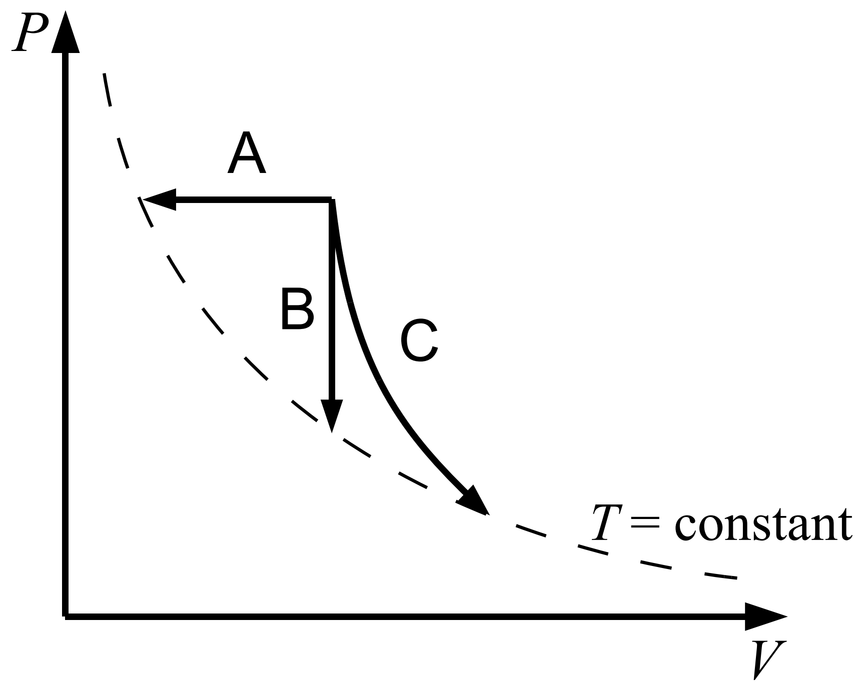
In both processes, the gas will do positive work, and hence lose energy through a work process.

For an isothermal process heat must be added to the gas to keep the energy (and hence temperature) constant.

For an adiabatic process, no heat can be added, so the energy (and hence temperature) of the gas must decrease.

Both processes start at the same energy and temperature, so the adiabatic process will end up at a lower temperature than the isothermal process.

Isothermal processes do not change a gas's internal energy. If n moles of an ideal gas are taken along the following paths, all ending at the same temperature, for which is the change in internal energy $\Delta U = nC_V\Delta T$?



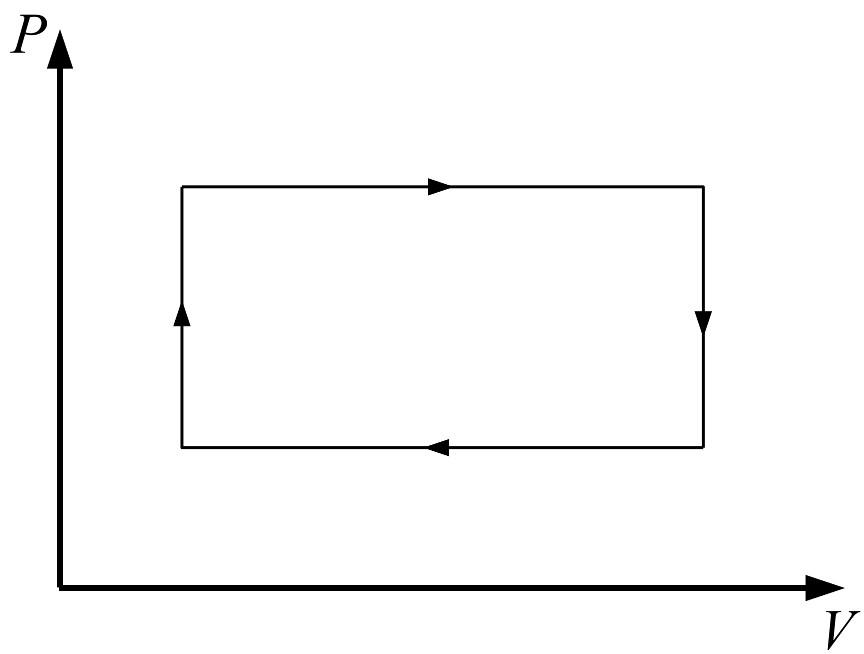
1. Path A (isobaric)
2. Path B (isochoric)
3. Path C (adiabatic)
4. All of the above
5. None of the above

ANS: **4**—The change in energy can always be expressed as $\Delta U = nC_V\Delta T$.

Remember, U is always proportional to T, so ΔU will be proportional to ΔT by the same factor, regardless of the process.

The symbol C_V is misleading here. We read it as "molar heat capacity in a constant-volume process," which it certainly is when we are discussing heat: $Q = nC_V\Delta T$ for an isochoric process. However, when relating ΔU to ΔT, we should not use the word heat capacity. It would be better to use the term "energy capacity." Therefore, while C_V is the molar *heat* capacity for a constant volume process, it is the molar *energy* capacity for all processes.

A gas is cycled by successive combinations of heating and cooling, compression and expansion as shown below.



Does the gas perform a net amount of work during a full cycle?

1. Yes, a positive amount
2. Yes, a negative amount
3. No, the net work performed is zero
4. Need more information

ANS: **1**—The gas does a positive amount of work in the cycle.

The gas does no work in the isochoric processes on the sides. It does negative work in the bottom (isobaric) process. Because the positive work was done at higher pressure, there is greater area under the top curve than under the bottom curve. The total work done in the process is the sum of the works (positive for the top, negative for the bottom), which is simply the area enclosed by the curve.

In a clockwise cycle, the area enclosed by the curve is the positive work done by the gas during the entire cycle.

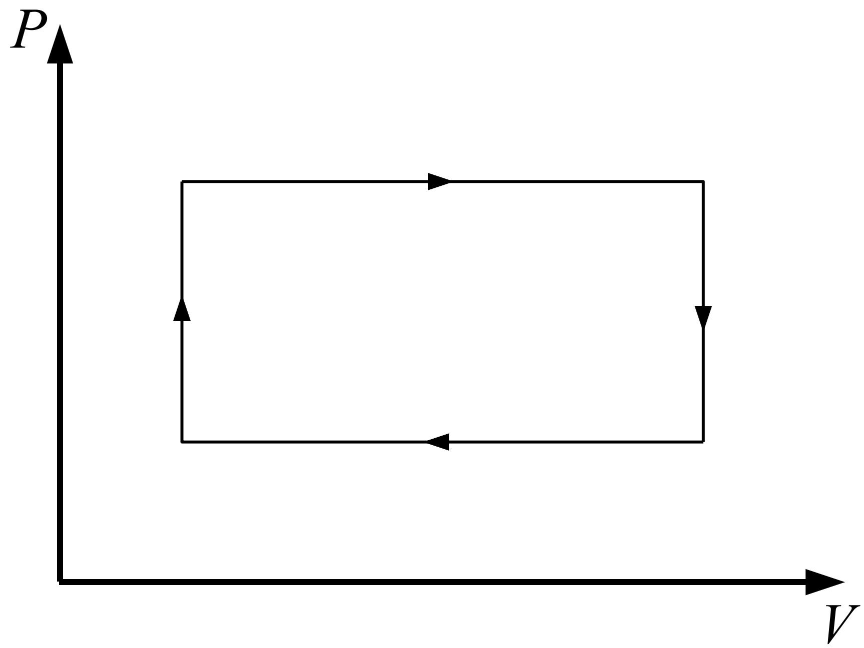
When an ideal gas goes through a thermodynamic process and returns to its original pressure and volume, which of the following must be true?

1. The net amount of heat absorbed must equal the net amount of heat emitted
2. The net amount of work done on the gas must equal the net amount of work done by the gas
3. The internal energy must be unchanged
4. all of the above

ANS: **3**—The internal energy is unchanged in any closed thermodynamic process (complete cycle).

The net amount of work done by the gas will only equal the net amount of work done on the gas if the cycle encloses no area. (Think of a process that takes the gas from state A to B, then right back to state A along the reverse of the original process.) The net amount of heat added will also only equal the net amount of heat emitted if the cycle encloses no area.

A gas is cycled by successive combinations of heating and cooling, compression and expansion as shown below.



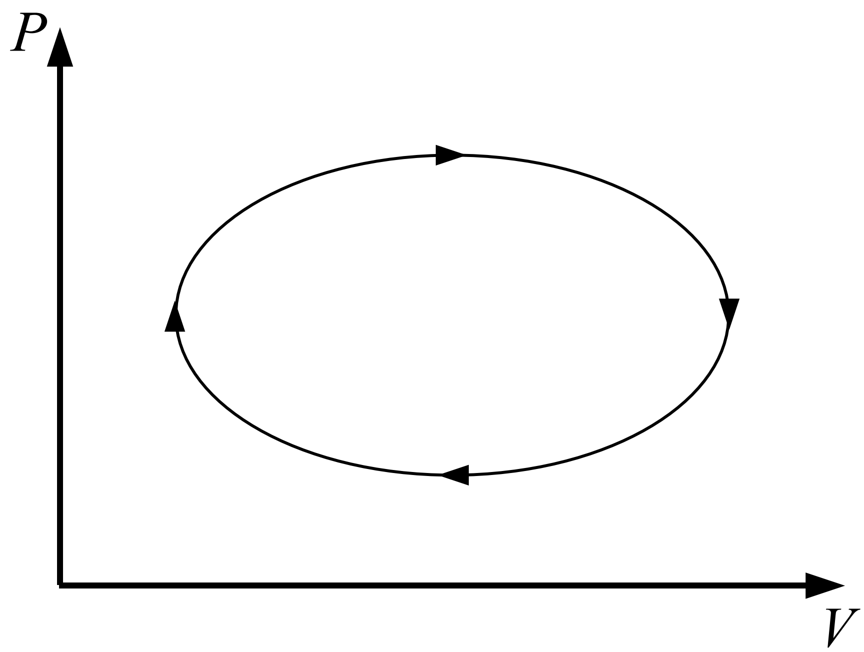
Does the gas absorb a net amount of heat during a full cycle?

1. Yes, a positive amount
2. Yes, a negative amount
3. No, the net heat absorbed is zero
4. Need more information

ANS: **1**—The gas absorbs a net positive amount of heat in the cycle.

The energy does not change over the entire cycle, so the heat added to the gas during the cycle must be equal to the work done by the gas in the cycle ($\Delta U = Q - W = 0$). The gas does a net positive amount of work, so it absorbs a net positive amount of heat.

A gas is contained in a cylinder with a movable piston. Its pressure and volume are “cycled,” i.e. always returning to the same state, according to the following graph:



Over the course of one complete cycle

1. a net amount of heat is added.
2. a net amount of heat is removed.
3. no heat is transferred.
4. heat is transferred, but the amount added is the same as the amount removed, so the net amount of heat added is zero.

ANS: **1**—A net amount of heat is added to the gas.

The cycle is clockwise, meaning that the gas does a net amount of work.

If you cannot see it, consider breaking the cycle up into two pieces. The first piece goes from the left-most point to the right-most point, following the top half of the ellipse. The area under the curve in this process is positive, so the gas does a positive amount of work during that process.

Next, consider the process that goes from the right-most point back to the left-most point. The area under the curve in this process is neative, so the gas does a negative amount of work during that process.

The area under the top process is greater in magnitude than the area under the second process, so the net work done over the full cycle is positive. As I mentioned earlier, the net work done in any clockwise cycle is positive. (The net work done in a counter-clockwise cycle is negative.)

Since $\Delta U = 0$ for the closed cycle, and the gas does a net positive amount of work, the gas must also absorb a net positive amount of heat.

In a power plant, burning fuel (oil, gas, or coal) is used to drive a heat engine. A given amount of fuel thus produces a specific amount of energy as mechanical work, which drives a generator and produces electricity. That electricity can be then used to power an electric heater in your home.

Burning that same amount of fuel directly within your home would produce

1. less heat
2. the same heat
3. more heat

than the electric heater produced.

Answer: 1, it produces less heat. The reason is that even an optimal heat engine cannot covert heat into work with perfect 100% efficiency. Some heat must be expelled to a cooler reservoir, and the net work equals the difference between the heat input and the heat output:

$$W_{net} = Q_{net} = Q_{input} - Q_{output}$$

Efficiency is defined as the ratio of the work output to the heat input

$$e = \frac{W_{net}}{Q_{input}} = Q_{net} = \frac{(Q_{input} - Q_{output})}{Q_{input}}$$

which must always be less than unity, so when the turbine work produces electrical energy, it's always less than Q_{input}. Burning the fuel directly would release the full Q_{input}.

Electric heaters are simple devices where the amount of electrical energy used yields the same amount of energy produced as heat:
input 10 J of electricity energy, output 10 J of heat.

Can any device take an input of 10 J of energy and get an output of more than 10 J of heat?

1. No, it would violate the first law of thermodynamics
2. Yes, doing 10 J of work can transfer more than 10 J of heat

Answer: 2, a given amount of energy as work can move a greater amount of energy as heat. Once again, net work equals the difference between the heat input and the heat output:

$$W_{net} = Q_{net} = Q_{input} - Q_{output}$$

Now, both net work done by the gas and net heat added to the gas are negative, meaning the environment does a positive amount of work on the gas and the gas expels more heat than it absorbs. This is what a heat pump does.

Efficiency is now measured by the ratio of the heat output to how much work it takes to transfer it. Because the net work and heat are negative, we use their magnitudes, writing

$$Q_{output} = Q_{input} - \ Q_{net} = Q_{output} = Q_{input} + \ |Q_{net}|$$

Such that

$$e = \frac{Q_{output}}{|W_{net}|} = \frac{Q_{input} + \ |Q_{net}|}{|Q_{net}|}$$

which must always be greater than unity, so when the heat pump does work to transfer heat, it's always more than $|W_{net}|$. Burning the fuel directly would release the full Q_{input}.

Warmup Question

A refrigerator works by absorbing heat from one region (the interior) and expelling it into another (typically behind it, where those dusty coils are). The trick is expanding a gas in contact with the interior so that it absorbs heat and compressing it in contact with the exterior so that it gives off heat.

Let's say you have an ideal refrigerator (whose engine works without friction) but a non-ideal roommate (who never works without coercion) who leaves the door ajar. Hence the distinction between "interior" and "exterior" is lost as air circulates freely between them. Can your refrigerator be good enough that it expels exactly the same amount of heat as it absorbs, thus not raising the temperature of the room? Make your argument using the first law of thermodynamics.

ANS: No. For energy to be conserved, the heat dumped into the room is equal to the heat taken from inside the refrigerator plus the work done by the refrigerator motor. As long as the fridge does work, it will add more heat to the room than it takes in, raising the temperature of the room.

Warmup Question

If a thermodynamic process is “closed,” i.e. the system returns to its original state at the end of a cycle, which of the following must be zero for the cycle?

1. The system’s net work (i.e. “work done by” minus “work done on”)
2. The net heat added to the system (i.e. “heat added” minus “heat removed”)
3. The net change in internal energy of the system
4. all of the above
5. none of the above

ANS: **3**—The net change in internal energy of the system is zero.

Energy is a state variable. It has a specific value at every point on a *PV* diagram. Since the cycle is closed, the final state is the same as the initial state, so the final energy is equal to the initial energy. Therefore, the internal energy does not change over the entire closed system, although it will change along various segments of the cycle.

The system's net work will be positive if the cycle is clockwise and encloses an area of the PV diagram. (It will be negative if it is counter-clockwise.) The net heat added to the system in a cycle is equal to the net work done by the gas.