MA207 Handout - Conditioning **Group Members:**_____

Review: The conditional probability of an event E occurring given that event F has occurred can be calculated in two ways. We can calculate $P(E|F)$ either by:

- Reducing the sample space to just the outcomes in F, OR
- Using the formula: $P(E|F) = \dfrac{P(EF)}{P(F)} = \dfrac{P(E \text{ and } F)}{P(F)}$.

When calculating the probability of an event, it can sometimes be useful to know if a related or precursor event occurs or not. Using a related event is called **conditioning** on that event, and the idea leads to the equation:

$$P(E) = P(E|F)\ P(F) + P(E|F^c)\ P(F^c).$$

Here F^c means F complement, or $NOT\ F$. We can generalize this equation as follows: Let $\{F_i\}$ be mutually exclusive events whose union is all of the sample space. Then

$$P(E) = \sum P(E|F_i)\ P(F_i).$$

Bayes's Formula uses these ideas to calculate a conditional probability by switching what is known and what is given. The simplest version of Bayes's Formula is:

$$P(E|F) = \frac{P(F|E)P(E)}{P(F)}.$$

This is a direct result of the formula version of calculating conditional probabilities followed by the multiplication rule: $P(EF) = P(F|E)P(E)$.

1. Robin is tied up over a steaming pool of lava while Two-Face laughs in the background. Two-face takes a coin out of his pocket to flip. He tells Robin that if the coin is heads, he will cut the rope and let Robin fall into the lava. If the coin is tails, Two-face will shoot robin with a gun. Batman overhears the conversation and predicts that he has a 3/4 chance of swinging in and catching Robin before he falls into the lave, but only a 2/3 chance of throwing his bat-a-rang to deflect the shot from Two-face's gun.

   (a) Assuming the coin is fair, what are the chances that Batman saves Robin?

   (b) Suppose that Two-face has a weighted coin which is twice as likely to be a heads as a tails. What are the chances that Two-face flips a heads?

   (c) Using the coin in part (b), what are the chances that Batman saves Robin?

   (d) Given that this is a TV show targeted at children, what are the chances that Batman saves Robin?

2. If Tom fixes dinner, Tammy will eat it. If Tom does not fix dinner, then Tammy will eat dinner only 1/2 the time. On any given night, Tom fixes dinner 2/3 of the time. What are the chances that Tammy will eat dinner?

3. A novice gladiator has to fight three battles (independent events). His probability of winning the first event, A_1, is $\frac{1}{2}$. The probability of winning the second event, A_2, is $\frac{1}{3}$ and the probability of winning the third event, A_3, is $\frac{1}{4}$. If he wins at least one event, he gets to compete in Rome.

   (a) What are the chances the gladiator loses all three matches?

   (b) What are the chances that the gladiator competes in Rome?

   (c) The gladiator competes in Rome. What is the probability that he won the first event?

4. Maria is playing with a set of three fair six-sided dice. Because she is six, she rolls the dice and puts aside any that turn up as a six. The remaining dice she then rerolls. She stops when all three dice are sixes. What are the chances that it takes two or fewer rolls for Maria to stop? (Hint: Condition on the number of sixes Maria rolls on the first roll.)

5. A sports team is scheduled to play three games in a tournament. The coach predicts the team has a 3/4 chance of winning the first match, a 1/2 chance to win the second match, and a 2/3 chance to win the third match. Since it is a double elimination tournament, they need to win at least two of the three to make it into the next round. Assume winning a match is independent of winning any other match.

   (a) What are the chances the team wins the first two matches?

   (b) What are the chances of the team making it into the next round?

   (c) Given the team makes it into the next round, what are the chances the team won the first match?

6. Dragon*Con is a science fiction / fantasy media convention in Atlanta that is held each year over Labor Day weekend. In 2019, over 80,000 fans attended the conference. People of all ages attend, and many dress in costume. Assume the following demo-graphical data for 2019:

Age range	% in costume	Percentage of conference attendance
$0-12$	90%	5%
$13-18$	75%	5%
$19-24$	80%	30%
$25-34$	60%	30%
$35-45$	40%	15%
$46-99$	30%	15%

   A randomly selected attendant is interviewed.

   (a) What are the chances the attendant is 13-18?

   (b) What are the chances the attendant is 13-18 years in age and in costume?

   (c) What are the chances the attendant is in costume?

   (d) Suppose the attendee is in costume, what are the chances that the selected individual is 13-18 years in age?