# Batman & Robin

a) $S$ = saves Robin

$H$ = coin is a heads.

$$P(S) = P(H)\,P(S|H) + P(H^c)\,P(S|H^c)$$
$$= \tfrac{1}{2}\left(\tfrac{3}{4}\right) + \tfrac{1}{2}\left(\tfrac{2}{3}\right)$$
$$= \tfrac{3}{8} + \tfrac{2}{6}$$
$$= \tfrac{9}{24} + \tfrac{8}{24}$$
$$= \tfrac{17}{24}$$

b) $P(H) = \tfrac{2}{3}$, $P(H^c) = \tfrac{1}{3}$ now.

$P(H) = 2P(T)$

$P(H) + P(T) = 1$

$\Rightarrow 2P(T) + P(T) = 1$

$P(T) = \tfrac{1}{3}$

c)

$$P(S) = \left(\tfrac{2}{3}\right)\left(\tfrac{3}{4}\right) + \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)$$
$$= \frac{6}{12} + \frac{2}{9}$$
$$= \frac{1}{2} + \frac{2}{9}$$
$$= \frac{13}{18}$$

d) 100%

# Tom & Tammy

D = Tom fixes dinner

E = Tammy eats dinner.

$P(D) = 2/3$

$P(E|D) = 1$

$P(E|D^c) = 1/2$

$$P(E) = P(E|D)P(D) + P(E|D^c)P(D^c)$$

$$= (1)(2/3) + (1/2)(1/3)$$

$$= \frac{2}{3} + \frac{1}{6}$$

$$= 5/6.$$

# Gladiator

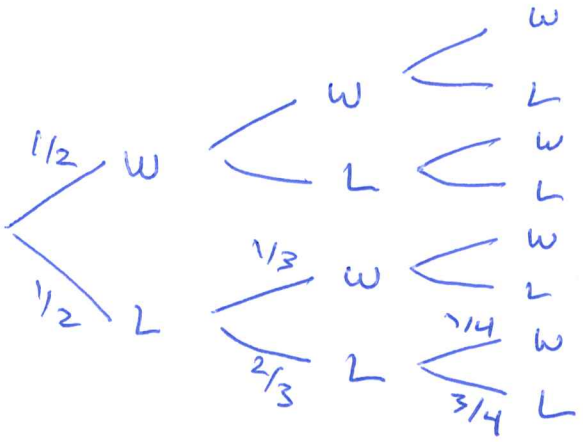
$P(A_1) = 1/2$

$P(A_2) = 1/3$

$P(A_3) = 1/4$

$A_i$ = winning event $i$.

$\Rightarrow A_i^c$ = losing event $i$




← losing all three!

a) $P(A_1^c A_2^c A_3^c) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1/4.$

b) $P(\text{compelling in Rome}) = P(\text{winning at least one}) = 3/4$

b/c $P(\text{winning at least one}) = 1 - P(\text{losing all 3}).$

c) $P(A_1 \mid \underbrace{\text{winning at least one}}_{\text{call this } W}) = P(A_1 \mid W)$

$= \dfrac{P(A_1 W)}{P(W)}$

$= \dfrac{P(W \mid A_1) P(A_1)}{P(W)} = \dfrac{(1)(1/2)}{3/4}$

$= 2/3$

Marra

Let $N$ = # of 6s on the 1st roll.

$E$ = stops in two rolls or less

$$P(E) = P(E \mid N=0)\,P(N=0) + P(E \mid N=1)\,P(N=1) + P(E \mid N=2)\,P(N=2) + P(E \mid N=3)\,P(N=3)$$

$$= \left(\frac{1}{216}\right)\left(\frac{125}{216}\right) + \left(\frac{1}{36}\right)\left(\frac{75}{216}\right) + \left(\frac{1}{6}\right)\left(\frac{15}{216}\right) + (1)\left(\frac{1}{216}\right)$$

get 2 sixes when she rolls two dice

gets a 6 when rolling one die.

$6^2 = 36$

$6^3 = 216$

$P(N=0) = \frac{5^3}{6^3} = \frac{125}{216}$

$P(N=1) = 3\left(\frac{25}{216}\right) = \frac{75}{216}$

$P(N=2) = 3\left(\frac{5}{216}\right) = \frac{15}{216}$

$P(N=3) = \frac{1}{216}$

# Soccer

let $W_i$ = winning $i^{th}$ match.

$P(W_1) = 3/4$ $\quad P(W_2) = 1/2$ $\quad P(W_3) = 2/3$.

Need at least two wins to get to the Next Round, call this NR.

a) $P(W_1 W_2) = (3/4)(1/2) = 3/8$ ← since events are independent.

b) $P(NR) = P(NR \mid W_1 W_2)\, P(W_1 W_2) + P(NR \mid W_1^c W_2)\, P(W_1^c W_2) +$
$P(NR \mid W_1 W_2^c)\, P(W_1 W_2^c) + P(NR \mid W_1^c W_2^c)\, P(W_1^c W_2^c)$

$= (1)(3/8) + (2/3)[(1/4)(1/2)] +$
$(2/3)[(3/4)(1/2)] + (0)[(1/4)(1/2)]$

$= 3/8 + \frac{2}{24} + \frac{6}{24} = 17/24$

c) $P(W_1 \mid NR) = \dfrac{P(NR \mid W_1)\, P(W_1)}{P(NR)}$

$= \dfrac{[1/2 + 1/2(2/3)](3/4)}{17/24}$

$= \dfrac{(5/6)(3/4)}{(17/24)} = \frac{15}{17}$

# Dragon * Con

let C = in costume

a) $P(13\text{-}18) = .05$ ← 5%.

b) $P(C \text{ and } 13\text{-}18) = P(13\text{-}18)\,P(C \mid 13\text{-}18)$

$= (.05)(.75) = .0375$

c) $P(C) = P(C \mid 0\text{-}12)\,P(0\text{-}12) +$
$P(C \mid 13\text{-}18)\,P(13\text{-}18) + \ldots +$
$P(C \mid 46\text{-}99)\,P(46\text{-}99)$

$= (.9)(.05) + (.75)(.05) + (.8)(.3) +$
$(.6)(.3) + (.4)(.15) + (.3)(.15)$

$= .6075$

d) $P(13\text{-}18 \mid C) = \dfrac{P(13\text{-}18 \text{ and } C)}{P(C)}$

$= \dfrac{P(C \mid 13\text{-}18)\,P(13\text{-}18)}{P(C)}$

$= \dfrac{(.05)(.75)}{.6075}$

$= .0617$