MA207 Sample Exam II

Name: KEY

Directions: You have one and a half hours to complete this exam. You may use your calculators, but no other electronic device. You may use the Ma207 Formula Sheet, t -distribution table, and standard normal distribution table provided, but no other notes or handouts. You may not use your book. Please turn off the ringer of your cellphone and store. Also, you must work alone.

I, Key**, am fully aware of and have abided by the BSC Honor Code in completing this exam.**

Please work in the space provided and *show all of your work.*

You can use!

1) A calculator

2) Standard Normal Table

3) formula sheet

4) T- Distribution table (which is not needed for test II)

1. While playing a game you are told to flip a fair coin. If you get a heads, you are asked to roll a fair six-sided die. If you get a tails, you are asked to roll a fair four-sided die.

   (a) (2 pts) What do the individual elements of the sample space look like?

   $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4\}$

   (b) (2 pts) What is the size of the sample space?

   $|S| = 10$

   (c) (2 pts) Are the outcomes equally likely? Why or why not?

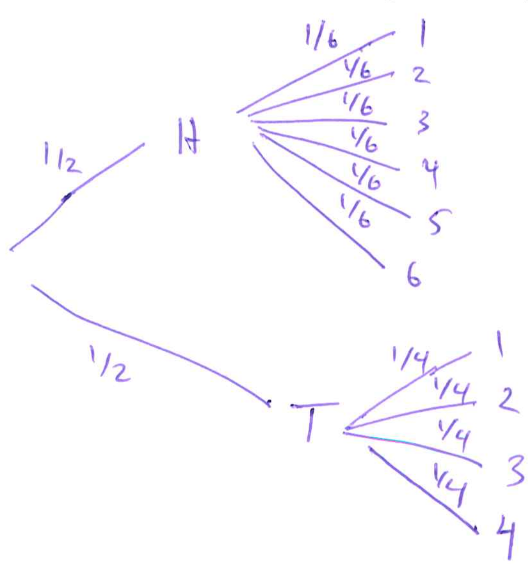
   No

   $P(H1) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

   $P(T1) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

   Not equal.

   (d) (6 pts) Build a tree diagram listing all possible outcomes.



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(e) (4 pts) What are the chances that you get a heads and roll a one?

$$P(H1) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

(f) (4 pts) What are the chances that you roll a one?

$$P(1) = P(1|H)P(H) + P(1|T)P(T)$$
$$= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \frac{1}{12} + \frac{1}{8}$$
$$= 5/24$$

(g) (4 pts) Let H be the event of getting a heads, and O the event of rolling a one. Are H and O independent? Why or why not?

No!

Not the same

$P(O) = 5/24 \leftarrow$ part (f)

$P(O|H) = 1/6 \leftarrow$ from problem

~~$P(H1) = \frac{1}{12} \leftarrow$ part (e)~~

(h) (4 pts) Given that you got a heads, what are the chances that you will roll a one?

$P(1|H) = 1/6 \leftarrow$ easy from problem. Here to compare to part (i)

(i) (4 pts) Given that you rolled a one, what are the chances that you had a heads?

$$P(H|1) = \frac{P(1|H)P(H)}{P(1)} = \frac{\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)}{5/24}$$
$$= \frac{\frac{1}{12}}{5/24} = \frac{1}{12} \cdot \frac{24}{5}$$
$$= \frac{2}{5}$$

used to have ~~three~~ components

2. The SAT is a standardized test used for college admissions. The mean on each of the ~~three~~ two components of the SAT is 500 with a standard deviation of 100. Each components is approximately normally distributed.

   (a) (4 pts) Suppose Linda Lovegood received a 710 on the Mathematics component of the SAT. What percentile did she achieve?

$$z = \frac{710-500}{100} = \frac{210}{100} = 2.10$$

Look up on SNT $\Rightarrow$ .9821

Linda Lovegood is at the 98th percentile.

   (b) (4 pts) Suppose Sam Sneed was at the 65 percentile on the Language component of the SAT. What score did Sam get for this component?

Look up z-score using .65 in the body of the SNT. We get

$$z \approx .385 = \frac{\text{score} - 500}{100}$$

$\Rightarrow$ Score = 538.5 or $\boxed{539}$

   (c) (4 pts) Oliver Overton's goal is to be at the 99th percentile on ~~all three~~ both components of the SAT. What is the minimum score Oliver can achieve on an individual component and still reach his goal?

z-score ≥ 2.33

$$2.33 = \frac{\text{Score} - 500}{100}$$

$$233 = \text{Score} - 500$$

$$\boxed{\text{Score} = 733}$$

3. Prostate cancer is the second most common cancer leading to death in men and the one most often diagnosed. About one in nine men will have prostate cancer in their lifetime. The table below indicates the odds of having prostate cancer based on age for men. (retrieved 7/9/19 from https://zerocancer.org/learn/about-prostate-cancer/facts-statistics/).

Age	Odds of having prostate cancer
Birth to 49	1 *in* 403
50 *to* 59	1 *in* 58
60 *to* 69	1 *in* 21
70+	1 *in* 12

The most common test for prostate cancer is the PSA test (a blood test) looking for elevated levels of a particular marker in the blood. Suppose that if you have prostate cancer, the PSA test will detect it 80% of the time (sensitivity). On the other hand, if you don't have prostate cancer, then the PSA test will show normal levels of the marker 90% of the time (specificity). (These percentages were made up). Also assume the PSA test is independent of the age of the patient whose blood is tested (which is likely).

(a) (4 pts) Given a randomly selected 56-year old male, what is the probability that he suffers from prostate cancer? (Hint: probabilities are decimals between 0 and 1).

$P(C \mid 56\text{-}59) = \frac{1}{58}$

(b) (4 pts) Given a randomly selected 56-year old male, what are the chances he gets a positive test result? (That is, a test result indicating prostate cancer.)

$$P(\text{pos} \mid 56\text{-}59) = P(\text{pos} \mid 56\text{-}59 + \text{canc})\,P(\text{can} \mid 56\text{-}59) + P(\text{pos} \mid 56\text{-}59 + \text{No Cancer})\,P(\text{No can} \mid 56\text{-}59)$$

$$= (.80)\left(\frac{1}{58}\right) + (.10)\left(\frac{57}{58}\right)$$

$$= .112$$

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(c) (4 pts) Suppose 56 year-old male's PSA blood test shows elevated levels of the marker (a postive test result), what is the probability he has prostate cancer?

$$P(C|pos) = \frac{P(pos|C)\,P(C)}{P(pos)}$$

$$= \frac{(.8)(1/58)}{.0112} = .123$$

(d) (2 pts) In Statville, the number of males of differing ages are provided in the table below:

Age	Number of males
Birth to 49	4000
50 *to* 59	500
60 *to* 69	300
70+	200

5000

Estimate the number of males in Statville who have prostate cancer.

~~P(C)~~ Number with cancer $= 4000\left(\frac{1}{403}\right) + 500\left(\frac{1}{58}\right) + 300\left(\frac{1}{21}\right) + 200\left(\frac{1}{12}\right)$

$= \cancel{49.49}\ 49.5$

(e) (2 pts) A randomly selected male from Statville is selected, what are the chances he has prostate cancer?

$$\frac{49.5}{5000} = .0099 \approx 1\%$$ chance.

(f) (4 pts) A male from Statville has prostate cancer. Calculate the probability the male is age 50 to 59.

$$P(50\text{-}59 | C) = \frac{P(C|50\text{-}59)\,P(50\text{-}59)}{P(C)}$$

$$= \frac{(1/58)\left(\frac{500}{5000}\right)}{.0099}$$

$$= .1741$$

4. Suppose a manufacturing process produces a usable microchip 90% of the time, independently of each other. A company packages and sells the microchips in batches of 11 chips.

(a) (4 pts) Let X be the number of usable microchips in a package. Explain why X is a binomial random variable.

1) # of trials is known, $n = 11$
2) Indepndent trials with same probability of success $p = .90$
3) we are counting the # of successes.

(b) (4 pts) Calculate the proportion of packages where all chips are usable, i.e. $P(X = 11)$.

$P(X=11) = (.9)^{11} = .3138$

(c) (6 pts) Calculate the proportion of packages where exactly 10 of the 11 chips are usable.

$P(X=10) = 11\ (.9)^{10}(.1) = .3835$

(d) (4 pts) Calculate the proportion of packages where at least 10 of the chips are usable.

$P(X \geq 10) = P(X=10) + P(X=11)$

$= .3835 + .3138 = .6973$

(e) (4 pts) A marketing consultant wants to claim that, "80% of all packages have ten or more usable microchips." Is the marketing consultant justified in making this claim? Why or why not?

No! ~~[illegible]~~ On average only 69.73% of packages have 10 or more useable chips.

5. On the isolated isle of Amazonia, the height of residents is normally distributed with a mean of 68 inches and a standard deviation of 6 inches. On the nearby isle of Aneaea the heights of residents is also normally distributed, but with a paltry mean of 62 inches and a standard deviation of 4 inches.

(a) (2 pts) A resident of Amazonia is chosen at random. What are the chances her height is at least 66 inches?

$P(66+ \mid \text{Amazonia}) = P(Z > -.33)$ From SNT

$z = \dfrac{66-68}{6} = -.33$ $\quad = .6293$

(b) (2 pts) A resident of Aneaea is chosen at random. What are the chances her height is at least 66 inches?

$P(66+ \mid \text{Aneaea}) = P(Z > 1) = .1587$ FROM SNT.

$z = \dfrac{66-62}{4} = 1$

(c) (4 pts) Suppose that the populations of Amazonia and Aneaea are roughly equal so that a person chosen at random would be equally likely to be from either island. A random individual is chosen. Calculate the probability her height is at least 66 inches.

Conditioning

$P(66+) = P(66+ \mid \text{Amazonia})\,P(\text{Amazonia}) + P(66+ \mid \text{Aneaea})\,P(\text{An})$

$= (.6293)(\tfrac{1}{2}) + (.1587)(\tfrac{1}{2}) = .3940$

(d) (4 pts) Suppose the population of Amazonia is twice that of Aneaea, so that a randomly selected individual is twice as likely to be from Amazonia as from Aneaea. A random individual is chosen. Calculate the probability her height is at least 66 inches.

$P(66+) = P(66+ \mid \text{Amazonia})\,P(\text{Amazon}) + P(66+ \mid \text{Aneaea})\,P(\text{Aneaea})$

$= \underbrace{(.6293)(\tfrac{2}{3}) + (.1587)(\tfrac{1}{3})} = .4724$

only change $P(\text{Amazon}) = 2P(\text{Aneaea})$

(e) (2 pts) Suppose the populations of Amazonia and Aneaea are roughly equal. A random individual is picked who height is at least 66 inches. Calculate the conditional probability that she is from Amazonia.

$P(\text{Amazonia} \mid 66+) = \dfrac{P(66+ \mid \text{Amazonia})\,P(\text{Amazonia})}{P(66+)}$

Answer to (a)

$= \dfrac{(.6293)(\tfrac{1}{2})}{.3940} = \boxed{.7986}$

Answer to (c)